MODULE -1

Introduction and Analysis of Members: Concept of Prestressing - Types of Prestressing - Advantages - Limitations –Prestressing systems - Anchoring devices - Materials - Mechanical Properties of high strength concrete - high strength steel - Stress-Strain curve for High strength concrete. Analysis of members at transfer - Stress concept - Comparison of behavior of reinforced concrete and prestressed concrete - Force concept - Load balancing concept - Kern point -Pressure line **10 Hours**

1**. Introduction This section covers the following topics.**

• Basic Concept

• Early Attempts of Prestressing

• Brief History

• Development of Building Materials

Basic Concept A prestressed concrete structure is different from a conventional reinforced concrete structure due to the application of an initial load on the structure prior to its use. The initial load or 'prestress' is applied to enable the structure to counteract the stresses arising during its service period. The prestressing of a structure is not the only instance of prestressing. The concept of prestressing existed before the applications in concrete. Two examples of prestressing before the development of prestressed concrete are provided. Force-fitting of metal bands on wooden barrels The metal bands induce a state of initial hoop compression, to counteract the hoop tension caused by filling of liquid in the barrels.

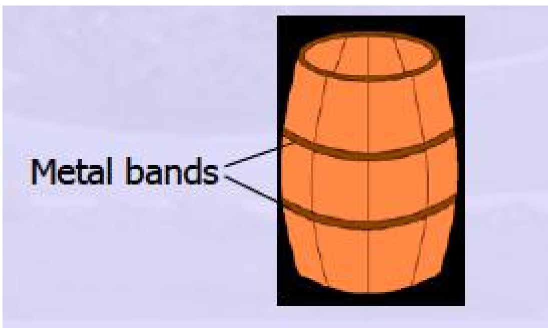


Figure 1-1.1 Force-fitting of metal bands on wooden barrels

Pre-tensioning the spokes in a bicycle wheel

The pre-tension of a spoke in a bicycle wheel is applied to such an extent that there will always be a residual tension in the spoke.

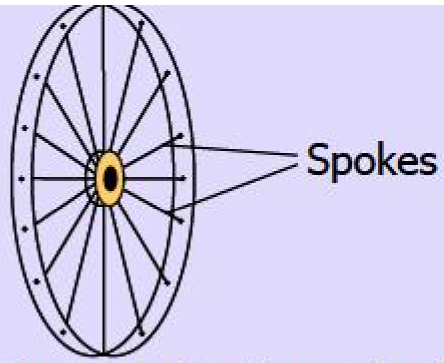


Figure 1-1.2 Pre-tensioning the spokes in a bicycle wheel

For concrete, internal stresses are induced (usually, by means of tensioned steel) for the following reasons.

- The tensile strength of concrete is only about 8% to 14% of its compressive strength.
- Cracks tend to develop at early stages of loading in flexural members such as beams and slabs.
- To prevent such cracks, compressive force can be suitably applied in the perpendicular direction.
- Prestressing enhances the bending, shear and torsional capacities of the flexural members.
- In pipes and liquid storage tanks, the hoop tensile stresses can be effectively counteracted by circular prestressing.

1.1.2 Early Attempts of Prestressing

Prestressing of structures was introduced in late nineteenth century. The following sketch explains the application of prestress.

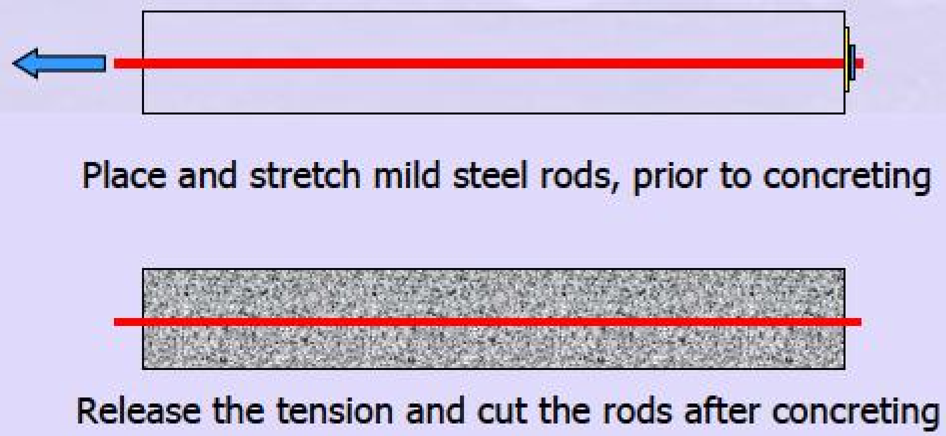


Figure 1-1.3 Prestressing of concrete beams by mild steel rods

Mild steel rods are stretched and concrete is poured around them. After hardening of concrete, the tension in the rods is released. The rods will try to regain their original length, but this is prevented by the surrounding concrete to which the steel is bonded. Thus, the concrete is now effectively in a state of pre-compression. It is capable of counteracting tensile stress, such as arising from the load shown in the following sketch.

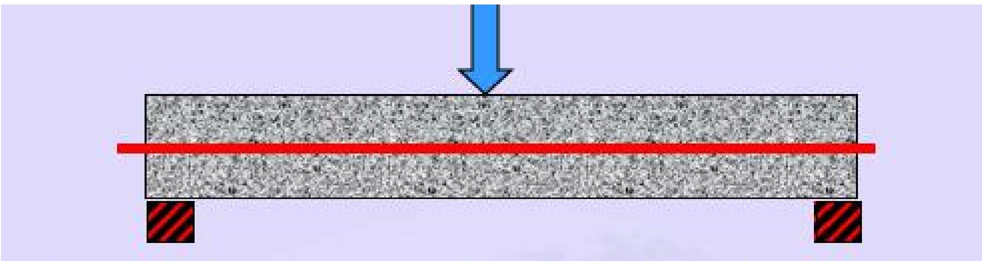


Figure 1-1.4 A prestressed beam under an external load

But, the early attempts of prestressing were not completely successful. It was observed that the effect of prestress reduced with time. The load resisting capacities of the members were limited. Under sustained loads, the members were found to fail. This was due to the following reason.

Concrete shrinks with time. Moreover under sustained load, the strain in concrete increases with increase in time. This is known as creep strain. The reduction in length due to **creep** and **shrinkage** is also applicable to the embedded steel, resulting in significant loss in the tensile strain.

In the early applications, the strength of the mild steel and the strain during prestressing were less. The residual strain and hence, the residual prestress was only about 10% of the initial value. The following sketches explain the phenomena.

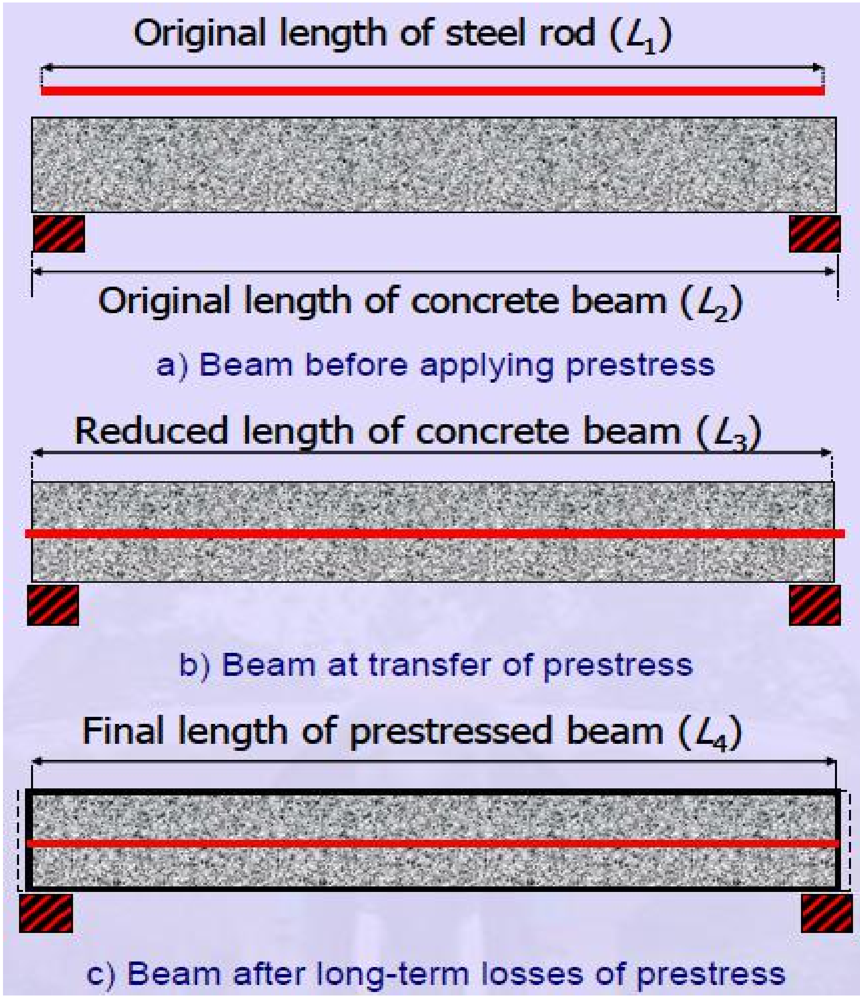


Figure 1-1.5 Variation of length in a prestressed beam

The residual strain in steel = original tensile strain in steel – compressive strains corresponding to short-term and long-term losses.
Original tensile strain in steel = $(L_2 - L_1)/L_1$
Compressive strain due to elastic shortening of beam = $(L_2 - L_3)/L_1$
(short-term loss in prestress)
Compressive strain due to creep and shrinkage = $(L_3 - L_4)/L_1$
(long-term losses in prestress)
Therefore, residual strain in steel = $(L_4 - L_1)/L_1$

The maximum original tensile strain in mild steel = Allowable stress / elastic modulus
$= 140$ MPa / 2×10^5 MPa
$= 0.0007$

The total loss in strain due to elastic shortening, creep and shrinkage was also close to 0.0007. Thus, the residual strain was negligible.
The solution to increase the residual strain and the effective prestress are as follows.
- Adopt **high strength steel** with much higher original strain. This leads to the scope of high prestressing force.
- Adopt **high strength concrete** to withstand the high prestressing force.

Advantages and Types of Prestressing
This section covers the following topics.
- Definitions
- Advantages of Prestressing
- Limitations of Prestressing
- Types of Prestressing

1.2.1 Definitions
The terms commonly used in prestressed concrete are explained. The terms are placed in groups as per usage.
Forms of Prestressing Steel
Wires
Prestressing wire is a single unit made of steel.
Strands
Two, three or seven wires are wound to form a prestressing strand.
Tendon
A group of strands or wires are wound to form a prestressing tendon.
Cable
A group of tendons form a prestressing cable.
Bars
A tendon can be made up of a single steel bar. The diameter of a bar is much larger than that of a wire.
The different types of prestressing steel are further explained in Section 1.7, Prestressing Steel.

Nature of Concrete-Steel Interface

Bonded tendon

When there is adequate bond between the prestressing tendon and concrete, it is called a bonded tendon. Pre-tensioned and grouted post-tensioned tendons are bonded tendons.

Stages of Loading

The analysis of prestressed members can be different for the different stages of loading. The stages of loading are as follows.

1) Initial : It can be subdivided into two stages.

a) During tensioning of steel

b) At transfer of prestress to concrete.

2) Intermediate : This includes the loads during transportation of the prestressed members.

3) Final : It can be subdivided into two stages.

a) At service, during operation.

b) At ultimate, during extreme events.

Advantages of Prestressing

The prestressing of concrete has several advantages as compared to traditional reinforced concrete (RC) without prestressing. A fully prestressed concrete member is usually subjected to compression during service life. This rectifies several deficiencies of concrete.

The following text broadly mentions the advantages of a prestressed concrete member with an equivalent RC member. For each effect, the benefits are listed.

1) Section remains uncracked under service loads

1. Reduction of steel corrosion
2. Increase in durability.
3. Full section is utilised
4. Higher moment of inertia (higher stiffness)
5. Less deformations (improved serviceability)
6. Increase in shear capacity.
7. Suitable for use in pressure vessels, liquid retaining structures.
8. Improved performance (resilience) under dynamic and fatigue loading

For the same span, less depth compared to RC member.

- Reduction in self-weight
- More aesthetic appeal due to slender sections
- More economical sections.

Types of Prestressing

Prestressing of concrete can be classified in several ways. The following classifications are discussed.

Source of prestressing force

This classification is based on the method by which the prestressing force is generated. There are four sources of prestressing force: Mechanical, hydraulic, electrical and chemical.

External or internal prestressing

This classification is based on the location of the prestressing tendon with respect to the concrete section.

Pre-tensioning or post-tensioning

This is the most important classification and is based on the sequence of casting the concrete and applying tension to the tendons.

Linear or circular prestressing

This classification is based on the shape of the member prestressed.

Full, limited or partial prestressing

Based on the amount of prestressing force, three types of prestressing are defined. **Uniaxial, biaxial or multi-axial prestressing**

As the names suggest, the classification is based on the directions of prestressing a member.
The individual types of prestressing are explained next.

Source of Prestressing Force

Hydraulic Prestressing

This is the simplest type of prestressing, producing large prestressing forces. The hydraulic jack used for the tensioning of tendons, comprises of calibrated pressure gauges which directly indicate the magnitude of force developed during the tensioning.

Mechanical Prestressing

In this type of prestressing, the devices includes weights with or without lever transmission, geared transmission in conjunction with pulley blocks, screw jacks with or without gear drives and wire-winding machines. This type of prestressing is adopted for mass scale production.

External or Internal Prestressing

External Prestressing

When the prestressing is achieved by elements located outside the concrete, it is called external prestressing. The tendons can lie outside the member (for example in I-girders or walls) or inside the hollow space of a box girder. This technique is adopted in bridges and strengthening of buildings. In the following figure, the box girder of a bridge is prestressed with tendons that lie outside the concrete.

Internal Prestressing

When the prestressing is achieved by elements located inside the concrete member (commonly, by embedded tendons), it is called internal prestressing. Most of the applications of prestressing are internal prestressing. In the following figure, concrete will be cast around the ducts for placing the tendons.

Pre-tensioning Systems and Devices

This section covers the following topics.

1. Introduction
2. Stages of Pre-tensioning
3. Advantages of Pre-tensioning
4. Disadvantages of Pre-tensioning
5. Devices
6. Manufacturing of Pre-tensioned Railway Sleepers

1. **Introduction**

Prestressing systems have developed over the years and various companies have patented their products. Detailed information of the systems is given in the product catalogues and brochures published by companies. There are general guidelines of prestressing in **Section 12** of **IS:1343 - 1980**. The information given in this section is introductory in nature, with emphasis on the basic concepts of the systems.

The prestressing systems and devices are described for the two types of prestressing, pre-tensioning and post-tensioning, separately. This section covers pre-tensioning. Section 1.4, "Post-tensioning Systems and Devices", covers post-tensioning. In pre-tensioning, the tension is applied to the tendons before casting of the concrete. The stages of pre-tensioning are described next.

2 Stages of Pre-tensioning

In pre-tensioning system, the high-strength steel tendons are pulled between two end abutments (also called bulkheads) prior to the casting of concrete. The abutments are fixed at the ends of a prestressing bed.

Once the concrete attains the desired strength for prestressing, the tendons are cut loose from the abutments.

The prestress is transferred to the concrete from the tendons, due to the bond between them. During the transfer of prestress, the member undergoes elastic shortening. If the tendons are located eccentrically, the member is likely to bend and deflect (camber).

The various stages of the pre-tensioning operation are summarised as follows.

1) Anchoring of tendons against the end abutments
2) Placing of jacks
3) Applying tension to the tendons
4) Casting of concrete
5) Cutting of the tendons.

During the cutting of the tendons, the prestress is transferred to the concrete with elastic shortening and camber of the member.

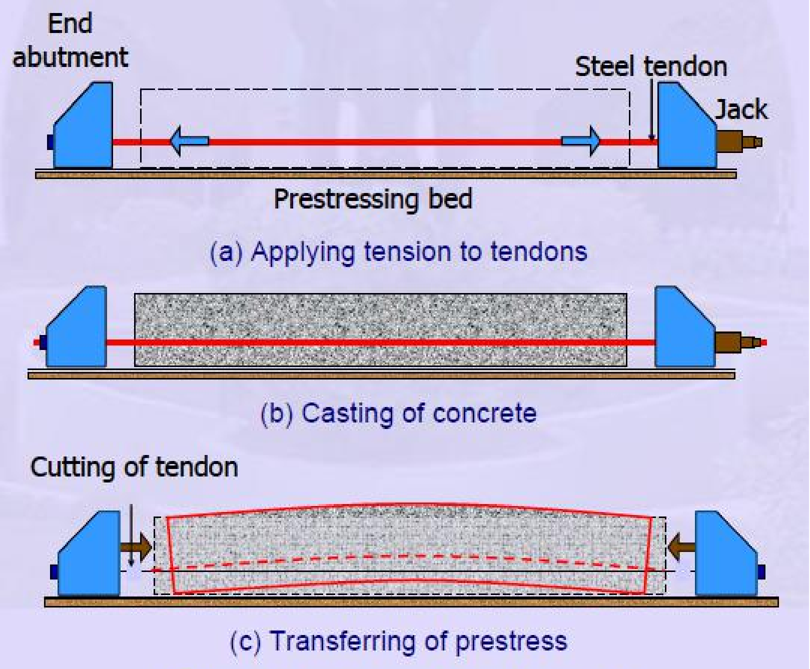


Figure1-3.1 Stages of pre-tensioning

1.3.3 Advantages of Pre-tensioning

The relative advantages of pre-tensioning as compared to post-tensioning are as follows.

• Pre-tensioning is suitable for precast members produced in bulk.

Prestressed Concrete Structures Dr. Amlan K Sengupta and Prof. Devdas Menon Indian Institute of Technology Madras

• In pre-tensioning large anchorage device is not present.

1.3.4 Disadvantages of Pre-tensioning

The relative disadvantages are as follows.

• A prestressing bed is required for the pre-tensioning operation.

• There is a waiting period in the prestressing bed, before the concrete attains sufficient strength.

• There should be good bond between concrete and steel over the transmission length.

Devices

The essential devices for pre-tensioning are as follows.

• Prestressing bed
• End abutments
• Shuttering / mould
• Jack
• Anchoring device

Analysis of Members under Flexure (Part I)

This section covers the following topics.

• Introduction

• Analyses at Transfer and at Service

Introduction

Similar to members under axial load, the analysis of members under flexure refers to the evaluation of the following.

1) Permissible prestress based on allowable stresses at transfer.
2) Stresses under service loads. These are compared with allowable stresses under service conditions.
3) Ultimate strength. This is compared with the demand under factored loads.
4) The entire load versus deformation behaviour.

The analyses at transfer and under service loads are presented in this section. The analysis for the ultimate strength is presented separately in Section 3.4, Analysis of Member under Flexure (Part III). The evaluation of the load versus deformation behaviour is required in special type of analysis. This analysis will not be covered in this section.

Assumptions

The analysis of members under flexure considers the following.

1) Plane sections remain plane till failure (known as Bernoulli's hypothesis).
2) Perfect bond between concrete and prestressing steel for bonded tendons.

Principles of Mechanics

The analysis involves three principles of mechanics.

1) **Equilibrium** of internal forces with the external loads. The compression in concrete (*C*) is equal to the tension in the tendon (*T*). The couple of C and T are equal to the moment due to external loads.
2) **Compatibility** of the strains in concrete and in steel for bonded tendons. The formulation also involves the first assumption of plane section remaining plane after bending. For unbonded tendons, the compatibility is in terms of deformation.
3) **Constitutive** relationships relating the stresses and the strains in the materials.

Variation of Internal Forces

In reinforced concrete members under flexure, the values of compression in concrete (*C*) and tension in the steel (*T*) increase with increasing external load. The change in the lever arm (*z*) is not large.

In prestressed concrete members under flexure, at transfer of prestress *C* is located close to *T*. The couple of *C* and *T* balance only the self weight. At service loads, *C* shifts up and the lever arm (*z*) gets large. The variation of *C* or *T* is not appreciable.

The following figure explains this difference schematically for a simply supported beam under uniform load.

The analyses at transfer and under service loads are similar. Hence, they are presented together. A prestressed member usually remains uncracked under service loads. The concrete and steel are treated as elastic materials. The principle of superposition is applied. The increase in stress in the prestressing steel due to bending is neglected.

There are three approaches to analyse a prestressed member at transfer and under service loads. These approaches are based on the following concepts.

a) Based on stress concept.

b) Based on force concept.
c) Based on load balancing concept.

The following material explains the three concepts.

Based on Stress Concept

In the approach based on stress concept, the stresses at the edges of the section under the internal forces in concrete are calculated. The stress concept is used to compare the calculated stresses with the allowable stresses.

The following figure shows a simply supported beam under a uniformly distributed load (UDL) and prestressed with constant eccentricity (*e*) along its length

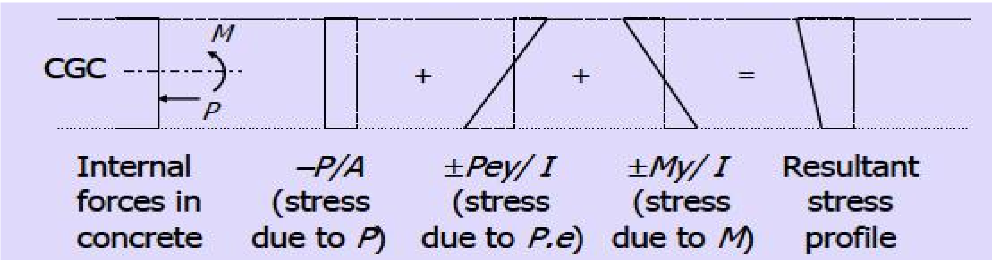


Figure 3-2.3 Stress profiles at a section due to internal forces

$$f = -\frac{P}{A} \pm \frac{Pey}{I} \pm \frac{My}{I}$$

Based on Force Concept

The approach based on force concept is analogous to the study of reinforced concrete. The tension in prestressing steel (*T*) and the resultant compression in concrete (*C*) are considered to balance the external loads. This approach is used to determine the dimensions of a section and to check the service load capacity. Of course, the stresses in concrete calculated by this approach are same as those calculated based on stress concept. The stresses at the extreme edges are compared with the allowable stresses

Based on Load Balancing Concept

The approach based on load balancing concept is used for a member with curved or harped tendons and in the analysis of indeterminate continuous beams. The moment, upward thrust and upward deflection (camber) due to the prestress in the tendons are calculated. The upward thrust balances part of the superimposed load

Example 1

A concrete beam prestressed with a parabolic tendon is shown in the figure. The prestressing force applied is 1620 kN. The uniformly distributed load includes the self weight. Compute the extreme fibre stress at the mid-span by applying the three concepts. Draw the stress distribution across the section at mid-span.

Solution

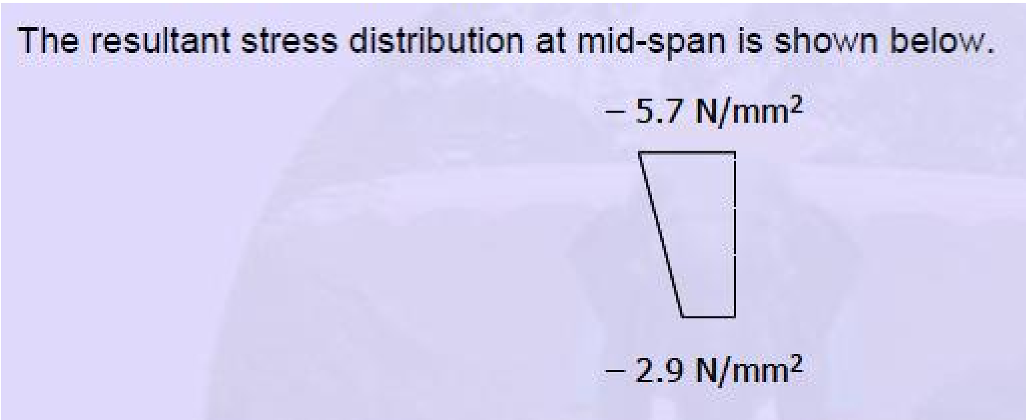
a) Stress concept

Area of concrete, $A = 500 \times 750$
$= 375{,}000\ mm^2$
Moment of inertia, $I = (500 \times 750^3) / 12$
$= 1.758 \times 10^{10}\ mm^4$
Bending moment at mid-span, $M = (45 \times 7.32) / 8$
= 299.7 kNm

Top fibre stress (C ,T) = - 4.32 + 5.01- 6.39 = - 5.7 N/mm

Bottom fibre stress (C, T) = -4.32 - 5.01+ 6.39 = -2.9 N/mm

Lever arm $z = M / P$
$= 299.7 \times 10^3 / 1620$
= 185 mm
Eccentricity of C $e_c = z - e$
= 185 – 145
= 40 mm



MODULE -2

Losses in Prestress, Loss of Prestress due to Elastic shortening, Friction, Anchorage slip, Creep of concrete, Shrinkage of concrete and Relaxation of steel - Total Loss. Deflection and Crack Width Calculations of Deflection due to gravity load. Deflection due to prestressing force -Total deflection - Limits of deflection - Limits of span-to-effective depth ratio - Calculation of Crack Width - Limits of crack width **10 hours**

Losses in Prestress

This section covers the following topics.

1. Introduction
2. Elastic Shortening

The relevant notations are explained first.

Notations

Geometric Properties

The commonly used geometric properties of a prestressed member are defined as follows.

A_c = Area of concrete section

= Net cross-sectional area of concrete excluding the area of prestressing steel.

A_p = Area of prestressing steel

= Total cross-sectional area of the tendons.

A = Area of prestressed member

= Gross cross-sectional area of prestressed member.

$= A_c + A_p$

A_t = Transformed area of prestressed member

= Area of the member when steel is substituted by an equivalent area of concrete.

$$= A_c + mA_p$$

$$= A + (m - 1)A_p$$

Here,

m = the modular ratio = E_p/E_c

E_c = short-term elastic modulus of elasticity.

The following figure shows the commonly used areas of the prestressed members.

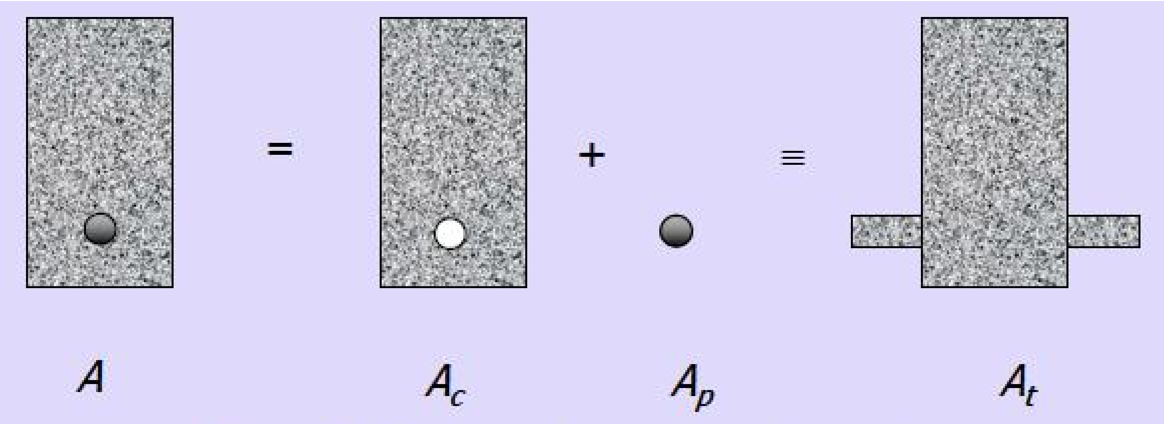


Figure 2-1.1 Areas for prestressed members

CGC = Centroid of concrete
= Centroid of the gross section. The CGC may lie outside the concrete (Figure 2-1.2).
CGS = Centroid of prestressing steel
= Centroid of the tendons. The CGS may lie outside the tendons
I = Moment of inertia of prestressed member
= Second moment of area of the gross section about the CGC.
I_t = Moment of inertia of transformed section
= Second moment of area of the transformed section about the centroid of the transformed section.
e = Eccentricity of CGS with respect to CGC

Load Variables

Pi = Initial prestressing force
= The force which is applied to the tendons by the jack.
*P*0 = Prestressing force after immediate losses
= The reduced value of prestressing force after elastic shortening, anchorage slip and loss due to friction.
Pe = Effective prestressing force after time-dependent losses
= The final value of prestressing force after the occurrence of creep, shrinkage and relaxation.

Introduction

In prestressed concrete applications, the most important variable is the prestressing force. In the early days, it was observed that the prestressing force does not stay constant, but reduces with time. Even during prestressing of the tendons and the transfer of prestress to the concrete member, there is a drop of the prestressing force from the recorded value in the jack gauge. The various reductions of the prestressing force are termed as the losses in prestress.

The losses are broadly classified into two groups, immediate and time-dependent. The immediate losses occur during prestressing of the tendons and the transfer of prestress to the concrete member. The time-dependent losses occur during the service life of the prestressed member. The losses due to elastic shortening of the member, friction at the tendon-concrete interface and slip of the anchorage are the immediate losses. The losses due to the shrinkage and creep of the concrete and relaxation of the steel are the time-dependent losses. The causes of the various losses in prestress are shown in the following chart.

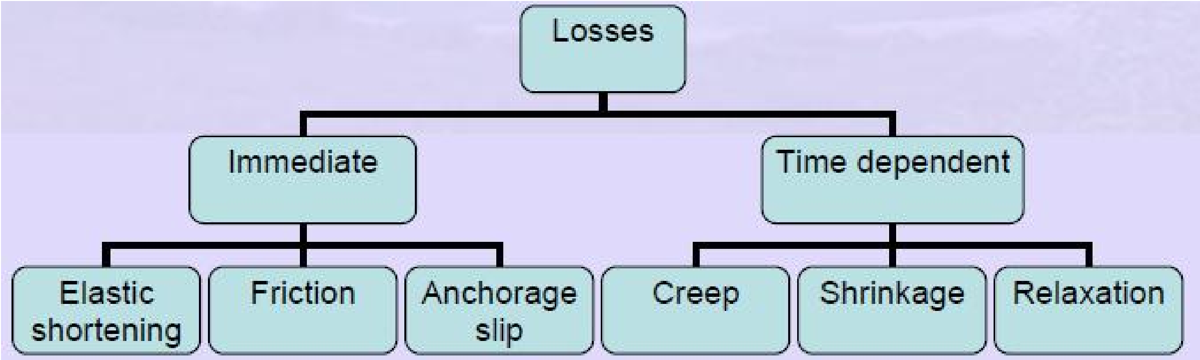


Figure 2-1.3 Causes of the various losses in prestress

Elastic Shortening

Pre-tensioned Members

When the tendons are cut and the prestressing force is transferred to the member, the concrete undergoes immediate shortening due to the prestress. The tendon also shortens by the same amount, which leads to the loss of prestress.

Post-tensioned Members

If there is only one tendon, there is no loss because the applied prestress is recorded after the elastic shortening of the member. For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons.

The elastic shortening loss is quantified by the drop in prestress (Δf_p) in a tendon due to the change in strain in the tendon ($\Delta \varepsilon_p$). It is assumed that the change in strain in the tendon is equal to the strain in concrete (ε_c) at the level of the tendon due to the prestressing force. This assumption is called **strain compatibility** between concrete and steel. The strain in concrete at the level of the tendon is calculated from the stress in concrete (f_c) at the same level due to the prestressing force. A linear elastic relationship is used to calculate the strain from the stress.

For simplicity, the loss in all the tendons can be calculated based on the stress in concrete at the level of CGS. This simplification cannot be used when tendons are stretched sequentially in a post-tensioned member. The calculation is illustrated for the following types of members separately.

1. Pre-tensioned Axial Members
2. Pre-tensioned Bending Members
3. Post-tensioned Axial Members
4. Post-tensioned Bending Members

1. Pre-tensioned Axial Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned axial member.

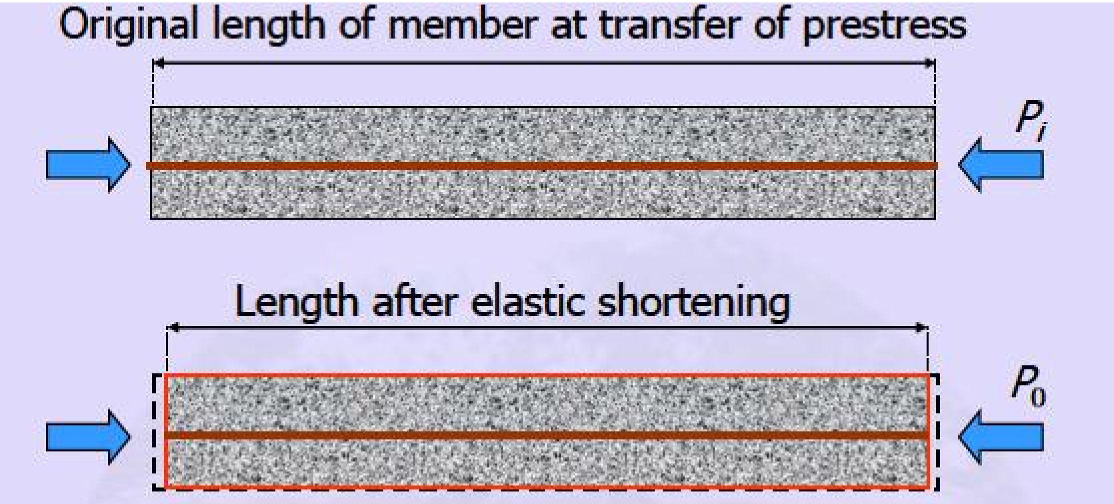


Figure 2-1.4 Elastic shortening of a pre-tensioned axial member

The loss can be calculated as per Eqn. (2-1.1) by expressing the stress in concrete in terms of the prestressing force and area of the section as follows.*t*
Δf =mf

Note that the stress in concrete due to the prestressing force after immediate losses (P_0/A_c) can be equated to the stress in the transformed section due to the initial prestress (P_i/A_t). This is derived below. Further, the transformed area A_t of the prestressed member can be approximated to the gross area A.
The following figure shows that the strain in concrete due to elastic shortening (ε_c) is the difference between the initial strain in steel (ε_{pi}) and the residual strain in steel (ε_{p0}).

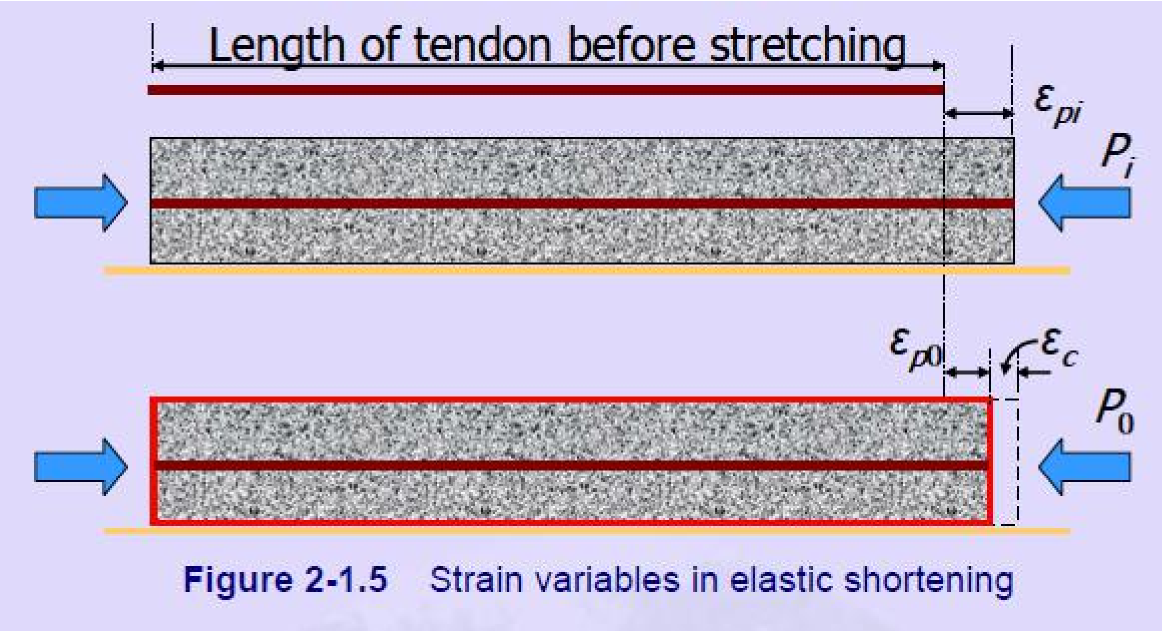


Figure 2-1.5 Strain variables in elastic shortening

The following equation relates the strain variables.

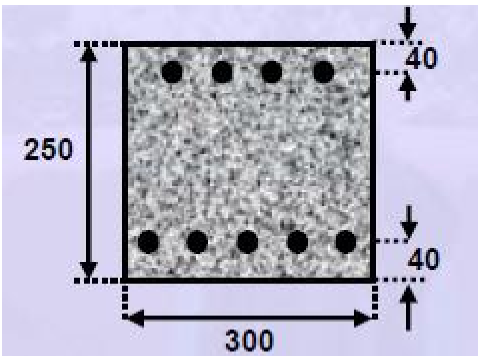
$\varepsilon_c = \varepsilon_{pi} - \varepsilon_{p0}$

The strains can be expressed in terms of the prestressing forces as follows.

$P\ \varepsilon = AE$

Example 2-1.1

A prestressed concrete sleeper produced by pre-tensioning method has a rectangular cross-section of 300mm × 250 mm (*b* × *h*). It is prestressed with 9 numbers of straight 7mm diameter wires at 0.8 times the ultimate strength of 1570 N/mm^2. Estimate the percentage loss of stress due to elastic shortening of concrete. Consider *m* = 6.



a) Approximate solution considering gross section

The sectional properties are calculated as follows.

Area of a single wire, $A_w = \pi/4 \times 7^2$

$= 38.48\ mm^2$

Area of total prestressing steel, $A_p = 9 \times 38.48$

$= 346.32\ mm^2$

Area of concrete section, $A = 300 \times 250$

$= 75 \times 10^3\ mm^2$

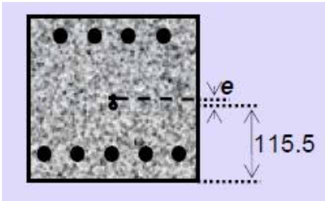
Moment of inertia of section, $I = 300 \times 250^3/12$

$= 3.91 \times 10^8$ mm4

Distance of centroid of steel area (CGS) from the soffit,

=(4×38.48×250- 40+5×38.48×40)

y = 9×38.48 =115.5 mm



Prestressing force, $P_i = 0.8 \times 1570 \times 346.32$ N

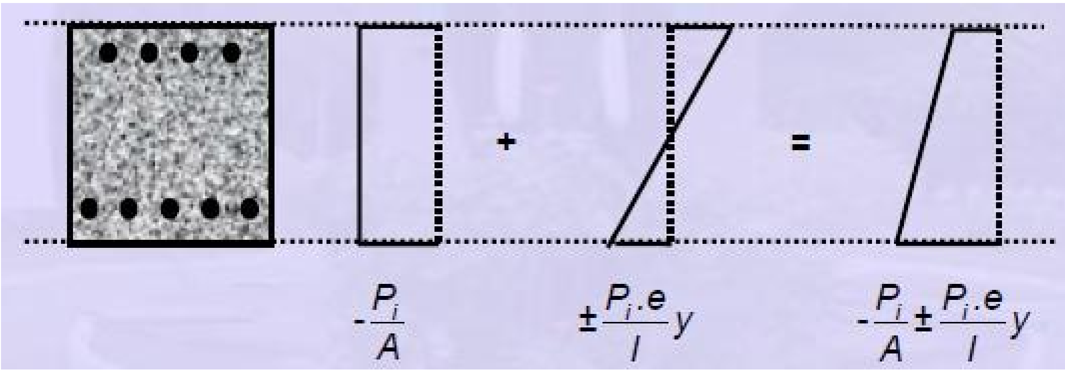
= 435 kN

Eccentricity of prestressing force,

$e = (250/2) - 115.5$

= 9.5 mm

The stress diagrams due to P_i are shown.



Since the wires are distributed above and below the CGC, the losses are calculated for the top and bottom wires separately.

Stress at level of top wires ($y = y_t = 125 - 40$)

Loss of prestress in top wires = *mfcAp*
(in terms of force) = 6 × 4.9 × (4 × 38.48)
= 4525.25 N
Loss of prestress in bottom wires = 6 × 6.7 × (5 × 38.48)
= 7734.48 N
Total loss of prestress = 4525 + 7735
= 12259.73 N
≈ 12.3 kN
Percentage loss = (12.3 / 435) × 100%
= 2.83%
b) Accurate solution considering transformed section.
Transformed area of top steel,
*A*1 = (6 – 1) 4 × 38.48
= 769.6 mm2
Transformed area of bottom steel,
*A*2 = (6 – 1) 5 × 38.48
= 962.0 mm2
Total area of transformed section,
AT = *A* + *A*1 + *A*2
= 75000.0 + 769.6 + 962.0
= 76731.6 mm2

Centroid of the section (CGC)
y = *A*× + *A* × + *A* ×*A*
125 1 (250- 40) 2 40
= 124.8 mm from soffit of beam
Moment of inertia of transformed section,
IT = *Ig* + *A*(0.2)2 + *A*1(210 – 124.8)2 + *A*2(124.8 – 40)2
= 4.02 × 108mm4
Eccentricity of prestressing force,
e = 124.8 – 115.5
= 9.3 mm
Stress at the level of bottom wires,
= - 435×10 - (435×10 ×9.3)84.8
76.73×10 4.02×10
= -5.67-0.85
= -6.52 N/mm
(fc)b
Stress at the level of top wires,
= - 435×10 + (435×10 ×9.3)85.2
76.73×10 4.02×10

= -5.67+0.86
= -4.81 N/mm
(fc)t
Loss of prestress in top wires = 6 × 4.81 × (4 × 38.48)
= 4442 N
Loss of prestress in bottom wires = 6 × 6.52 × (5 × 38.48)
= 7527 N
Total loss = 4442 + 7527
= 11969 N
≈ 12 kN
Percentage loss = (12 / 435) × 100%
= 2.75 %

Pre-tensioned Bending Members

The following figure shows the changes in length and the prestressing force due to elastic shortening of a pre-tensioned bending member

Due to the effect of self-weight, the stress in concrete varies along length (Figure 2-1.6). The loss can be calculated by Eqn. (2-1.1) with a suitable evaluation of the stress in concrete. To have a conservative estimate of the loss, the maximum stress at the level of CGS at the mid-span is considered.

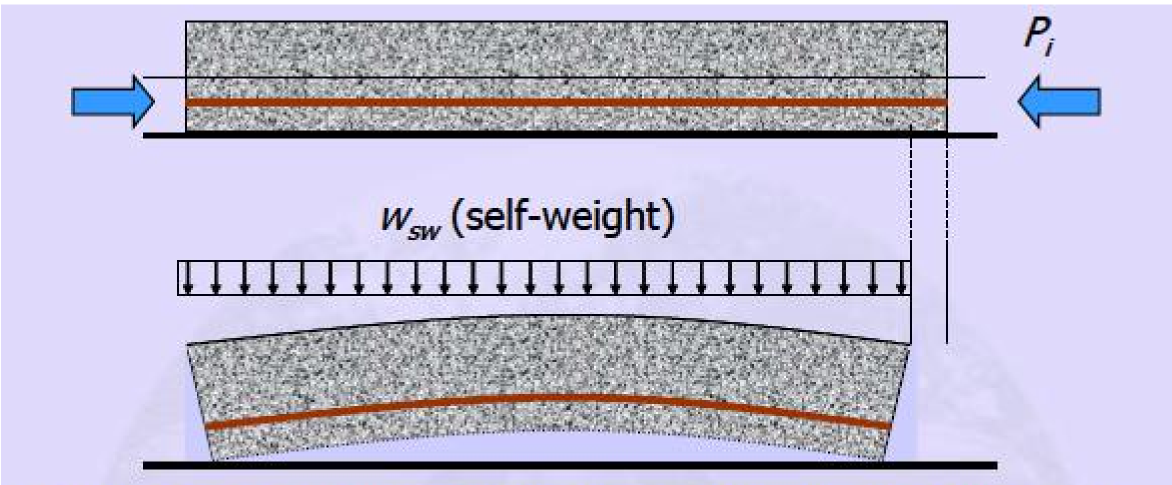


Figure 2-1.6 Elastic shortening of a pre-tensioned bending member

Post-tensioned Axial Members

For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons. The loss in each tendon can be calculated in progressive sequence. Else, an approximation can be used to calculate the losses.

The loss in the first tendon is evaluated precisely and half of that value is used as an average loss for all the tendons. Σ *p pcn i,j j= Δf = Δf mf*

Post-tensioned Bending Members

The calculation of loss for tendons stretched sequentially, is similar to post-tensioned axial members. For curved profiles, the eccentricity of the CGS and hence, the stress in concrete at the level of CGS vary along the length. An average stress in concrete can be considered.

Losses in Prestress (Part II)

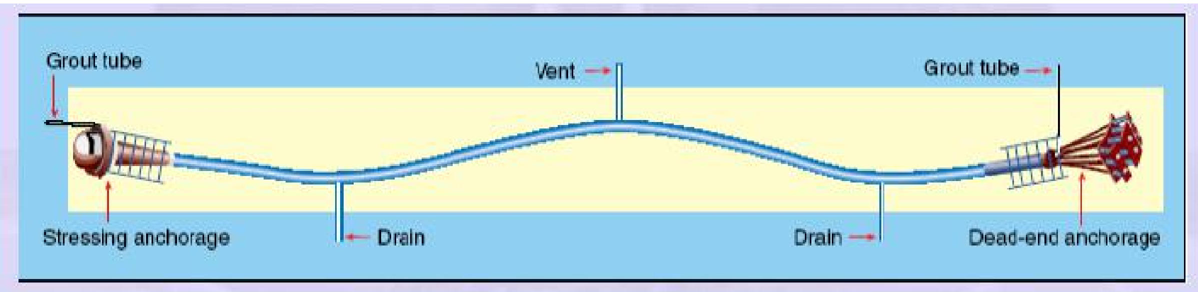
This section covers the following topics

- **Friction**
- **Anchorage Slip**
- **Force Variation Diagram**

Friction

The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member, leads to a drop in the prestress along the member from the stretching end. The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons.

The friction is generated due to the curvature of the tendon and the vertical component of the prestressing force. The following figure shows a typical profile (laying pattern) of the tendon in a continuous beam.



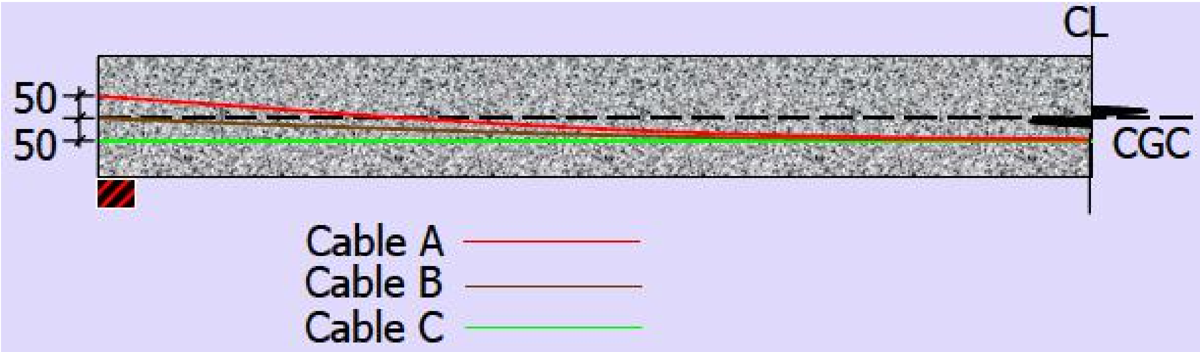
In addition to friction, the stretching has to overcome the **wobble** of the tendon. The wobble refers to the change in position of the tendon along the duct. The losses due to friction and wobble are grouped together under friction.

The derivation of the expression of *P* is based on a circular profile. Although a cable profile is parabolic based on the bending moment diagram, the error induced is insignificant.
The friction is proportional to the following variables.

1. Coefficient of friction (μ) between concrete and steel.
2. The resultant of the vertical reaction from the concrete on the tendon (N) generated due to curvature.

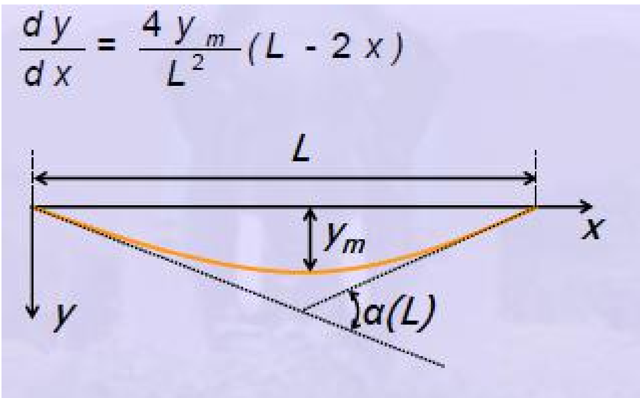
Example 2-2.1

A post-tensioned beam 100 mm × 300 mm ($b \times h$) spanning over 10 m is stressed by successive tensioning and anchoring of 3 cables A, B, and C respectively as shown in figure. Each cable has cross section area of 200 mm^2 and has initial stress of 1200 MPa. If the cables are tensioned from one end, estimate the percentage loss in each cable due to friction at the anchored end. Assume μ = 0.35, k = 0.0015 / m.



Solution

Prestress in each tendon at stretching end = 1200 × 200
= 240 kN.
To know the value of *α(L),* the equation for a parabolic profile is required.



y_m = displacement of the CGS at the centre of the beam from the ends
L = length of the beam
x = distance from the stretching end
y = displacement of the CGS at distance x from the ends.

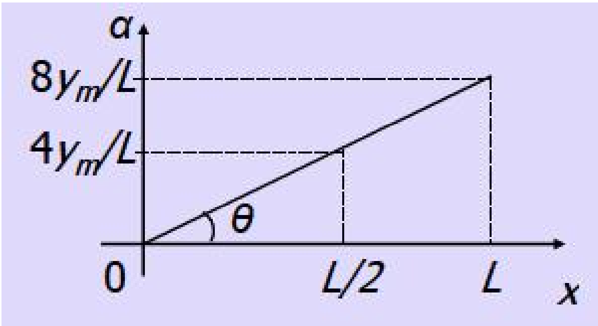
An expression of $\alpha(x)$ can be derived from the change in slope of the profile. The slope of the profile is given as follows.

$$\frac{dy}{dx} = \frac{4y_m}{L^2}(L - 2x)$$

At $x = 0$, the slope $dy/dx = 4y_m/L$. The change in slope $\alpha(x)$ is proportional to x.

The expression of $\alpha(x)$ can be written in terms of x as $\alpha(x) = \theta.x$,

where, $\theta = 8y_m/L^2$. The variation is shown in the following sketch.

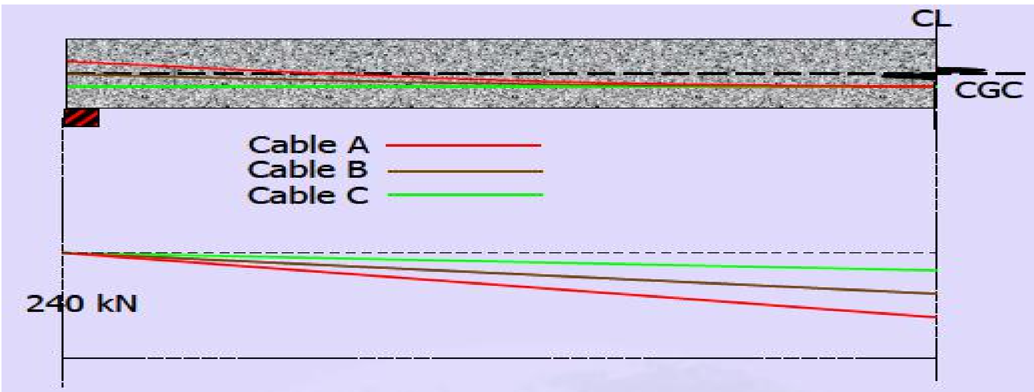


The total subtended angle over the length L is $8y_m/L$.

The prestressing force P_x at a distance x is given by

$$P_x = P_0 e^{-(\mu\alpha + kx)} = P_0 e^{-\eta x}$$

$$\eta x = \mu\alpha + kx$$



Variation of prestressing forces

The loss due to friction can be considerable for long tendons in continuous beams with changes in curvature. The drop in the prestress is higher around the intermediate supports where the curvature is high. The remedy to reduce the loss is to apply the stretching force from both ends of the member in stages.

2-2.2 Anchorage Slip

In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space. The anchorage block also moves before it settles on the concrete. There is loss of prestress due to the consequent reduction in the length of the tendon.

The total anchorage slip depends on the type of anchorage system. In absence of manufacturer's data, the following typical values for some systems can be usedDue to the setting of the anchorage block, as the tendon shortens, there is a reverse friction. Hence, the effect of anchorage slip is present up to a certain length (Figure 2- 2.4). Beyond this **setting length**, the effect is absent. This length is denoted as l_{set}.

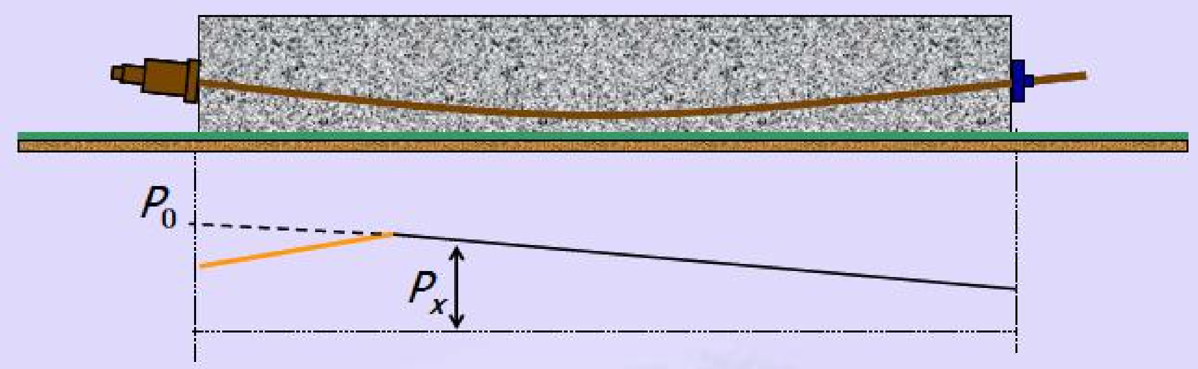


Figure 2-2.4 Variation of prestressing force after anchorage slip

Force Variation Diagram

The magnitude of the prestressing force varies along the length of a post-tensioned member due to friction losses and setting of the anchorage block. The diagram representing the variation of prestressing force is called the force variation diagram.

Considering the effect of friction, the magnitude of the prestressing force at a distance *x* from the stretching end is given as follows.

P= *Pe* 0

Here, $\eta x = \mu\alpha + kx$ denotes the total effect of friction and wobble. The plot of P_x gives the force variation diagram.

The initial part of the force variation diagram, up to length l_{set} is influenced by the setting of the anchorage block. Let the drop in the prestressing force at the stretching end be ΔP. The determination of ΔP and l_{set} are necessary to plot the force variation diagram including the effect of the setting of the anchorage block.

Losses in Prestress (Part III)

This section covers the following topics.

- Creep of Concrete
- Shrinkage of Concrete
- Relaxation of Steel
- Total Time Dependent Losses

2.3.1 Creep of Concrete

Creep of concrete is defined as the increase in deformation with time under constant load. Due to the creep of concrete, the prestress in the tendon is reduced with time.

The creep of concrete is explained in Section 1.6, Concrete (Part II). Here, the information is summarised. For stress in concrete less than one-third of the characteristic strength, the ultimate creep strain ($\varepsilon_{cr,ult}$) is found to be proportional to the elastic strain (ε_{el}). The ratio of the ultimate creep strain to the elastic strain is defined as the ultimate creep coefficient or simply creep coefficient θ.

The following considerations are applicable for calculating the loss of prestress due to creep.

1) The creep is due to the sustained (permanently applied) loads only.

Temporary loads are not considered in the calculation of creep.

2) Since the prestress may vary along the length of the member, an average value of the prestress can be considered.

3) The prestress changes due to creep and the creep is related to the instantaneous prestress. To consider this interaction, the calculation of creep can be iterated over small time steps.

Shrinkage of Concrete

Shrinkage of concrete is defined as the contraction due to loss of moisture. Due to the shrinkage of concrete, the prestress in the tendon is reduced with time. The shrinkage of concrete was explained in details in the Section 1.6, Concrete (Part II).

IS:1343 - 1980 gives guidelines to estimate the shrinkage strain in **Section 5.2.4**. It is a simplified estimate of the ultimate shrinkage strain (ε_{sh}). Curing the concrete adequately and delaying the application of load provide long term benefits with regards to durability and loss of prestress. In special situations detailed calculations may be necessary to monitor shrinkage strain with time. Specialised literature or international codes can provide guidelines for such calculations.

Relaxation of Steel

Relaxation of steel is defined as the decrease in stress with time under constant strain.

Due to the relaxation of steel, the prestress in the tendon is reduced with time. The relaxation depends on the type of steel, initial prestress (f_{pi}) and the temperature. To

Prestressed Concrete Structures Dr. Amlan K Sengupta and Prof. Devdas Menon

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calculate the drop (or loss) in prestress (Δf_p), the recommendations of **IS:1343 - 1980** can be followed in absence of test data.

MODULE-3

Design of Sections for Flexure: Analysis of members at ultimate strength - Preliminary Design - Final Design for Type 1members

Analysis of Members under Flexure Introduction Similar to members under axial load, the analysis of members under flexure refers to the evaluation of the following.

1) Permissible prestress based on allowable stresses at transfer.

2) Stresses under service loads. These are compared with allowable stresses underservice conditions.

3) Ultimate strength. This is compared with the demand under factored loads.

4) The entire load versus deformation behaviour.

The analyses at transfer and under service loads are presented in this section. The evaluation of the load versus deformation behaviour is required in special type of analysis. Assumptions The analysis of members under flexure considers the following.

1. Concrete is a homogeneous elastic material.

2. Within the range of working stress, both concrete & steel behave elastically, notwithstanding the small amount of creep, which occurs in both the materials under the sustained loading.

3. A plane section before bending is assumed to remain plane even after bending, which implies a linear strain distribution across the depth of the member.

4. Prestress Concrete is one in which there have been introduced internal stresses of such magnitude and distribution that stresses resulting from given external loading is counter balanced to a desired degree.

5. Plane sections remain plane till failure (known as Bernoulli’s hypothesis).

6. Perfect bond between concrete and prestressing steel for bonded tendons.

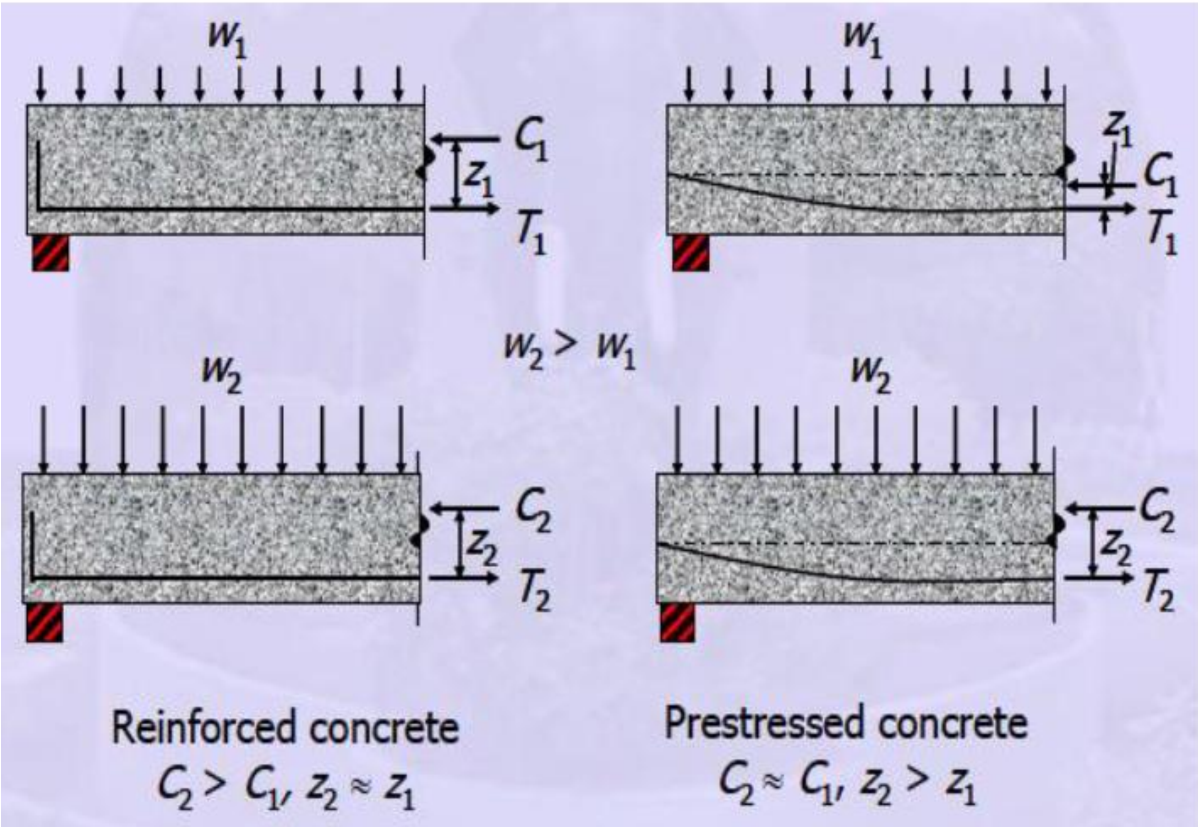
Principles of Mechanics The analysis involve three principles of mechanics.

1) Equilibrium of internal forces with the external loads. The compression in concrete (C) is equal to the tension in the tendon (T). The couple of C and T are equal to the moment due to external loads.

2) Compatibility of the strains in concrete and in steel for bonded tendons. The formulation also involves the first assumption of plane section remaining plane after bending. For unbonded tendons, the compatibility is in terms of deformation.

3) Constitutive relationships relating the stresses and the strains in the materials. Variation of Internal Forces In reinforced concrete members under flexure, the values of compression in concrete (C) and tension in the steel (T) increase with increasing external load. The change in the lever arm (z) is not large

In prestressed concrete members under flexure, t transfer of prestress C is located close to T. The couple of C and T balance only the self-weight. At service loads, C shifts up and the lever arm (z) gets large. The variation of C or T is not appreciable. The following figure explains this difference schematically for a simply supported beam under uniform load.



*C*1, *T*1 = compression and tension at transfer due to self-weight
*C*2, *T*2 = compression and tension under service loads
*w*1 = self-weight
*w*2 = service loads
*z*1 = lever arm at transfer
*z*2 = lever arm under service loads.

Analysis at Transfer and at Service

The analysis at transfer and under service loads is similar. Hence, they are presented together. A prestressed member usually remains uncracked under service loads. The concrete and steel are treated as elastic materials. The principle of superposition is applied. The increase in stress in the prestressing steel due to bending is neglected.

There are three approaches to analyse a prestressed member at transfer and under service loads. These approaches are based on the following concepts.

a) Based on stress concept.

b) Based on force concept.

c) Based on load balancing concept.

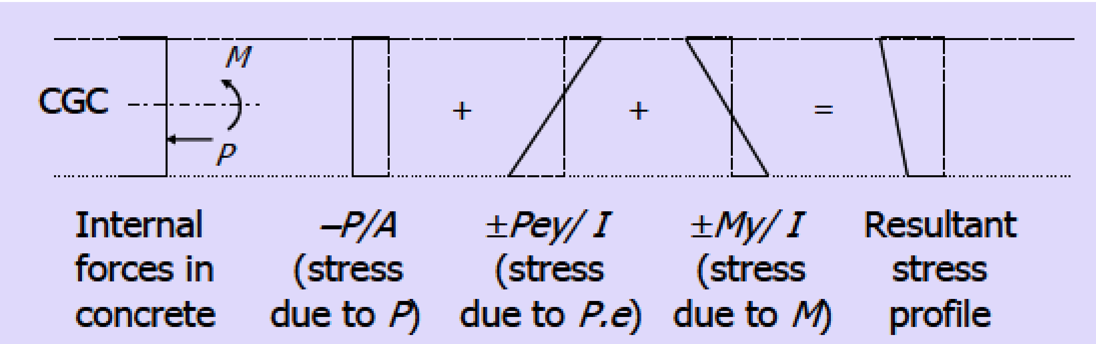
The following material explains the three concepts.

Based on Stress Concept

In the approach based on stress concept, the stresses at the edges of the section under the internal forces in concrete are calculated. The stress concept is used to compare the calculated stresses with the allowable stresses.

The following figure shows a simply supported beam under a uniformly distributed load (UDL) and prestressed with constant eccentricity (*e*) along its length.

The first stress profile is due to the compression *P*. The second profile is due to the eccentricity of the compression. The third profile is due to the moment. At transfer, the moment is due to self-weight. At service the moment is due to service loads.



Stress profiles at a section due to internal forces

The resultant stress at a distance *y* from the CGC is given by the principle of superposition as follows.

EXAMPLES

1. A posttensioned prestressed concrete beam having a rectangular section 150mm x 350mm has an effective cover of 50mm. if fck = 40 N/mm2, fp = 1600N/mm2 and the area of prestressing steel is 461 mm2, estimate the flexural strength of th section using (IS 1343) Indian standard code provisions.

Solution:

Given data

fck = 40 N/mm2

fp = 1600 N/mm2

AP = 461 N/mm2

D = d+d' = 300mm

The effective ratio is calculated as (refer IS 1343-1980,TABLE-1)

(fpxAp)/(fckxbxd) = (1600x461)/(40x150x300) = 0.40

From table 11, the corresponding value of (pre-tensioning)

(fpu/0.87fy)= 0.9

Xu/d = 0.783

(Apxfp)/(bxdxfck) =0.40

Fpu = 0.87x0.9xfp

Xu = 0.783d

fpu = (0.87x0.9x1600)

=1252.8 N/mm2

Xu = (0.783x300)

=234.9mm

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1252.8x461)x(300-0.42x234.9)

=116.28x10^6 N-mm

=116.28KN-m

2. A bonded prestressed concrete beam of rectangular cross section (200x500)mm . the tendon consists of 500mm2 area and it is stress to 1500N/mm2. The tendons are located at 100mm from the soffit of the beam. The concrete characteristics strength is 40 N/mm2 and Ec = 35KN/mm2 and Es = 200KN/mm2. Use IS 1343 code to calculate the ultimate strength.

Solution:

Given data

fck = 40 N/mm2

fp = 1500 N/mm2

AP = 500 N/mm2

D = d+d' = 500mm

The effective ratio is calculated as (refer IS 1343-1980,TABLE-1)

(fpxAp)/(fckxbxd) = (1500x500)/(200x400x40) = 0.23

From table 11, the corresponding value of (pre-tensioning)

(fpu/0.87fy)= 1.0

Xu/d = 0.5

(Apxfp)/(bxdxfck) =0.40

Fpu = 0.87x0.9xfp

Xu = 0.5d

fpu = (0.87x0.9x1500)

=1305 N/mm2

Xu = (0.5x400)

=200

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1305x500)x(400-0.42x250)

=192.48x10^6 N-mm

=192.48 KN-m

3. A pre tensioned beam of rectangular section 400mmx 1000mm overall depth is prestressed by 800 mm2of high tensile steel wires at an eccentricity of 300mm. if fck = 40N/mm2, fp = 1600N/mm2, estimate the ultimate flexural strength of the section. As per IS 1343 codal provisions.

Solution:

Given data

fck = 40 N/mm2

fp = 1600 N/mm2

AP = 461 N/mm2

D = d+d' = 1000 mm

The effective ratio is calculated as (refer IS 1343-1980,TABLE-1)

(fpxAp)/(fckxbxd) = (1600x800)/(40x400x800) = 0.1

From table 11, the corresponding value of (pre-tensioning)

(fpu/0.87fp)= 1.0

Xu/d = 0.217

(Apxfp)/(bxdxfck) =0.1

Fpu = 0.87x0.9xfp

Xu = 0.217d

fpu = (0.87x1x1600)

=1392 N/mm2

Xu = (0.217x800)

=173.6 mm

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1392x800)x(800-0.42x173.6)

=809.68x10^6 N-mm

=809.68 KN-m

4. Find the ultimate moment of resistance of the prestressed beam of the posttensioned beam section of width 300mm and effective depth is 600mm for the following data.

Fck = N/mm2, fp = 1500 N/mm2, Ap= 500mm2, fpu = 0.87x fy

Solution:

Given data

fck = 40 N/mm2

fp = 1500 N/mm2

AP = 500 N/mm2

D = d+d' = 600mm

The effective ratio is calculated as (refer IS 1343-1980,TABLE-1)

(fpxAp)/(fckxbxd) = (1500x500)/(40x600x300) = 0.104

Xu/d = 0.217

(Apxfp)/(bxdxfck) =0.104

Fpu = 0.87xfp

Xu = 0.217d

fpu = (0.87x1500)

=1305 N/mm2

Xu = (0.217x600)

=130.2mm

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1305x500)x(600-0.42x130.2)

=355.82x10^6 N-mm

=355.82KN-m

5. The cross section of a composite beam is of T-section having a pretensioned rib, 80 mm wide and 240 mm deep, and in situ cast slab, 350 mm wide and 80 mm thick. The pretensioned beam is reinforced with eight wires of 5 mm diameter with an ultimate tensile strength of 1600 N/mm2, located 60 mm from the soffit of the beam. The compressive strength of concrete in the in situ cast and precast elements is 20 and 40N/mm2, respectively. If the adequate reinforcements are provided to prevent shear failure at the intereface, estimate the flexural strength of the composite section.

Solution:

Given data

fck = 20 N/mm2

fp = 1600 N/mm2

Ap = 20 x 8 = 160 N/mm2

d = 240 mm

b = 350 mm

The effective reinforcement ratio is given by (IS 1343-1980,Table-1)

(fpxAp)/(fckxbxd) = (1600x160)/(20x350x240) = 0.152

Xu/d = 0.326

(fpu)/(0.87fp) =1

Fpu = 0.87xfp

Xu = 0.326d

fpu = (0.87x1600)

=1392 N/mm2

Xu = (0.326x240)

=78mm

Now refer IS 1343

M =Mu = fpuxAp (d-0.42Xu)

= (1392x160)x(240-0.42x78)

=46.15 x10^6 N-mm

=46.15KN-m

6. Find the ultimate moment of resistance of unbounded beam section of width 300 mm and effective depth 600 mm for the following data.
Fck = 40 n/mm2, fp=1500N/mm2, Ap=500mm2, length = 10m.

Solution:

Given data

fck = 40 N/mm2, fp = 1500 N/mm2, Ap = 500 N/mm2, d = 600 mm

b = 300 mm, l=10m

The effective reinforcement ratio is given by (IS 1343-1980,Table-1)

(fpxAp)/(fckxbxd) = (1500x500)/(40x300x600) = 0.1

l/d = 10x10^3/600 = 16.67

By iteration

l/d=20	fpu/fp=1.26
l/d=16.67	fpu/fp=?
l/d=10	fpu/fp=1.45

Fpu=1.32 x 1500 =1980N/mm2

Similarly

l/d=20 → Xu/d=0.32

l/d=16.67 →Xu/d =?

l/d=10 →Xu/d =0.36

Xu/d=0.33

Xu=0.33 x 600

=198mm

Moment of resistance,Mu = fpuxAp (d-0.42Xu)

= (1980x500)x(600-0.42x198)

=511.67 x10^6 N-mm

=511.67KN-m

7. A post tensioned beam with unbounded tendons is of rectangular section 400 mm wide with an effective depth of 800 mm. The cross sectional area of the prestressing steel is 2840 mm2. The effective prestressing steel after all the losses is 900N/mm2. The effective span of the beam is 16m. If fck=40N/mm2, estimate the ultimate moment of resistance of the section using IS1343.

Solution:

Given data

fck = 40 N/mm2

fp = 900 N/mm2

Ap = 2840 N/mm2

d = 8600 mm

b = 400 mm

l=16m

The effective reinforcement ratio is given by (IS 1343-1980,Table-1)

(fpxAp)/(fckxbxd) = (2840x900)/(40x400x800) = 0.199=0.2

l/d = 16x10^3/800 = 20

Fpu/fp=1.16

Fpu=1.16 x 900

=1044N/mm2

Xu/d=0.58

Xu=0.58 x 800

=464mm

Moment of resistance,Mu = fpuxAp (d-0.42Xu)

= (1044x2840)x(800-0.42x464)

=1794.15 x10^6 N-mm

=11794.15KN-m

MODULE-4

Design for Shear: Analysis for shear - Components of shear resistance - Modes of Failure - Limit State of collapse for shear - Design of transverse reinforcement.

INTRODUCTION

1. The behaviour of prestressed beams at failure in shear is distinctly different from their behaviour in flexure.

2. The beam will tend to fail abruptly without sufficient warning and the diagonal cracks that develop are considerably wider than the flexural cracks.

3. Shear forces result in shear stress. Such a stress can result in principal tensile stresses at the critical section which can exceed the tensile strength of the concrete

4. When the tensile strength of the concrete is exceeded cracks will formed. Cracking Patterns & Failure Modes

5. Cracking in prestressed concrete beams at ultimate load depends on the local magnitudes of moment and shear.

6. In regions where the moment is large and shear is small, vertical flexural cracks appear after the normal tensile stress in the extreme concrete fibres exceeds the tensile strength of concrete. This type of cracks shown as type A in figure.

7. Where both the moment and shear force are relatively large, flexural cracks which are vertical at the extreme fibres become inclined as they extend deeper into the beam owing to the presence of shear stresses in the beam web. These inclined cracks, which are often quite flat in a prestressed beam are called flexure-shear cracks and are designated crack type B. Cracking Patterns & Failure Modes

8. If adequate shear reinforcement is not provided, a flexure-shear crack may lead to a so-called shear-compression failure, in which the area of concrete in compression above the advancing inclined crack is so reduced as to be no longer adequate to carry compression force resulting from flexure.

9. Another type of inclined crack sometimes occurs in the web of a prestressed beam in the regions where moment is small and shear is large, such as the cracks designated type C adjacent to discontinuous support and near the point of contra flexure in the figure.

10. In such location, high principal tensile stress may cause inclined cracking in the mid-depth region of the beam before flexural cracking occurs in the extreme fibres. These cracks are known as web-shear cracks and occur most often in beams with relatively thin webs.

The analysis of reinforced concrete and prestressed concrete members for shear is more difficult compared to the analyses for axial load or flexure.

The analysis for axial load and flexure are based on the following principles of mechanics.

1) **Equilibrium** of internal and external forces

2) **Compatibility** of strains in concrete and steel

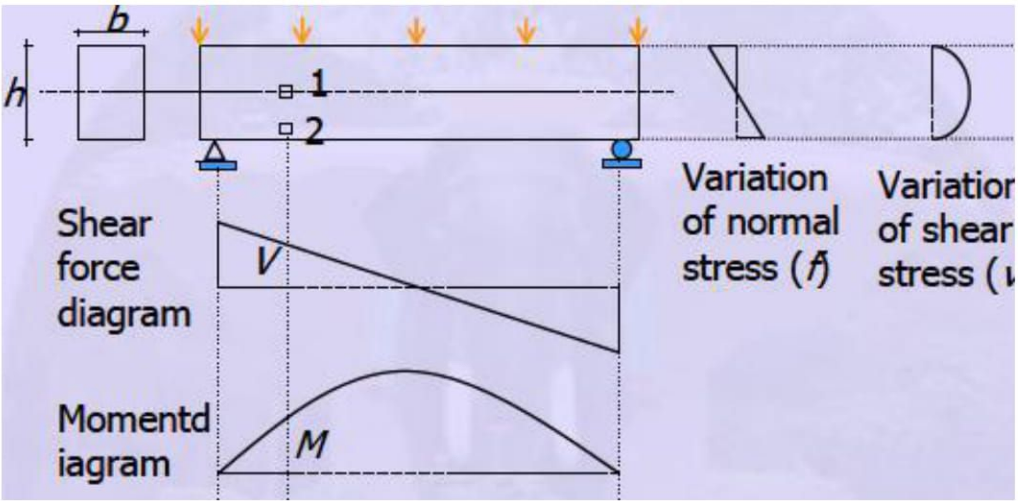
3) **Constitutive relationships** of materials.

The conventional analysis for shear is based on equilibrium of forces by a simple equation. The compatibility of strains is not considered. The constitutive relationships (relating stress and strain) of the materials, concrete or steel, are not used. The strength of each material corresponds to the ultimate strength. The strength of concrete under shear although based on test results, is empirical in nature.

Shear stresses generate in beams due to bending or twisting. The two types of shear stress are called flexural shear stress and torsional shear stress, respectively.

Stresses in an Uncracked Beam

The following figure shows the variations of shear and moment along the span of a simply supported beam under a uniformly distributed load. The variations of normal stress and shear stress along the depth of a section of the beam are also shown.



Variations of forces and stresses in a simply supported beam

Under a general loading, the shear force and the moment vary along the length. The normal stress and the shear stress vary along the length, as well as along the depth. The combination of the normal and shear stresses generate a two-dimensional stress field at a point. At any point in the beam, the state of two-dimensional stresses can be expressed in terms of the principal stresses. The Mohr's circle of stress is helpful to understand the state of stress.

Before cracking, the stress carried by steel is negligible. When the principal tensile stress exceeds the cracking stress, the concrete cracks and there is redistribution of stresses between concrete and steel. For a point on the neutral axis (Element 1), the shear stress is maximum and the normal stress is zero. The principal tensile stress ($\sigma 1$) is inclined at 45º to the neutral axis. The following figure shows the state of in-plane stresses.

Calculation of Shear Demand:

The objective of design is to provide ultimate resistance for shear (VuR) greater than the shear demand under ultimate loads (Vu). For simply supported prestressed beams, the maximum shear near the support is given by the beam theory. For continuous prestressed beams, a rigorous analysis can be done by the moment distribution method. Else, the shear coefficients in Table 13 of IS:456 - 2000 can be used under conditions of uniform cross-section of the beams, uniform loads and similar lengths of span.

Design of Stirrups

The design is done for the critical section. The critical section is defined in Clause 22.6.2 of IS:456 - 2000. In general cases, the face of the support is considered as the critical section. When the reaction at the support introduces compression at the end of the beam, the critical section can be selected at a distance effective depth from the face of the support. The effective depth is selected as the greater of dp or ds.

dp = depth of CGS from the extreme compression fiber

ds = depth of centroid of non-prestressed steel.

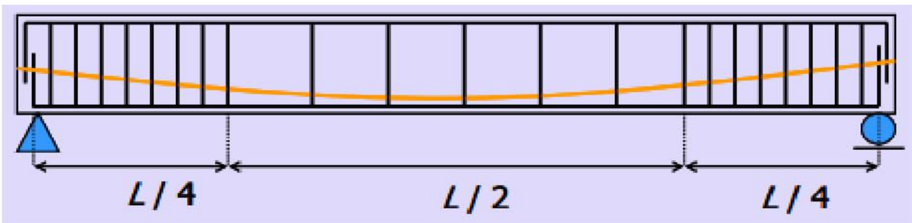
Since the CGS is at a higher location near the support, the effective depth will be equal to ds.

To vary the spacing of stirrups along the span, other sections may be selected for design. Usually the following scheme is selected for beams under uniform load.

1) Close spacing for quarter of the span adjacent to the supports.

2) Wide spacing for half of the span at the middle.

For large beams, more variation of spacing may be selected. The following sketch shows the typical variation of spacing of stirrups. The span is represented by L.



Typical variation of spacing of stirrup

Design of Transverse Reinforcement :

When the shear demand (Vu) exceeds the shear capacity of concrete (Vc), transverse reinforcements in the form of stirrups are required. The stirrups resist the propagation of diagonal cracks, thus checking diagonal tension failure and shear tension failure. The stirrups resist a failure due to shear by several ways.

The functions of stirrups are listed below.

1) Stirrups resist part of the applied shear.

2) They restrict the growth of diagonal cracks.

3) The stirrups counteract widening of the diagonal cracks, thus maintaining aggregate interlock to a certain extent.

4) The splitting of concrete cover is restrained by the stirrups, by reducing dowel forces in the longitudinal bars.

After cracking, the beam is viewed as a plane truss. The top chord and the diagonals are made of concrete struts. The bottom chord and the verticals are made of steel reinforcement ties. Based on this truss analogy, for the ultimate limit state, the total area of the legs of the stirrups (Asv) is given as follows.

$$\frac{A_{sv}}{s_v} = \frac{V_u - V_c}{0.87 f_y d_t}$$

The notations in the above equation are explained.

sv = spacing of the stirrups 48

dp = depth of CGS from the extreme compression fiber

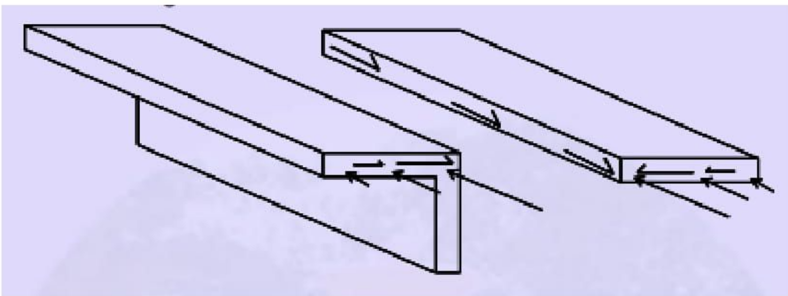
ds = depth of centroid of non-prestressed steel

fy = yield stress of the stirrups

The grade of steel for stirrups should be restricted to Fe 415 or lower.

Design of Stirrups for Flanges

For flanged sections, although the web carries the vertical shear stress, there is shear stress in the flanges due to the effect of shear lag. Horizontal reinforcement in the form of single leg or closed stirrups is provided in the flanges. The following figure shows the shear stress in the flange at the face of the web.



Shear stress in flange due to shear lag effect

The horizontal reinforcement is calculated based on the shear force in the flange. The relevant quantities for the calculation based on an elastic analysis are as follows.

1) Shear flow (shear stress × width)

2) Variation of shear stress in a flange (τf)

3) Shear forces in flanges (Vf).

4) Ultimate vertical shear force (Vu)

The following sketch shows the above quantities for an I-section (with flanges of constant widths).

The design shear force in a flange is given as follows.

Here,

bf = breadth of the flange

Df = depth of the flange

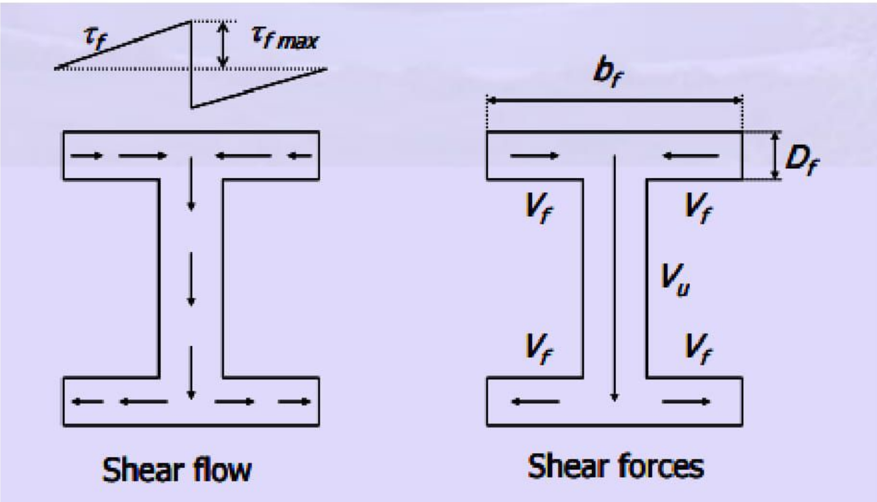


Figure 5-3.2 Shear flow and shear forces in an I-section

The design shear force in a flange is given as follows.

$$V_f = \frac{\tau_{f\,max}}{2}\frac{b_f}{2}D_f$$

Here,

bf = breadth of the flange

Df = depth of the flange

τf,max= maximum shear stress in the flange.

The maximum shear stress in the flange is given by an expression similar to that for the shear stress in web

$$\tau_{f\,max} = \frac{V_u A_1 \bar{y}}{I D_f}$$

Here,

Vu = ultimate vertical shear force

I = moment of inertia of the section.

A1 = area of half of the flange

= distance of centroid of half of the flange from the neutral axis at CGC.

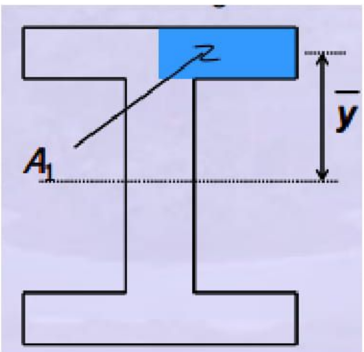


Figure : Cross-section of a beam showing the variables for calculating shear stress in the flange

The amount of horizontal reinforcement in the flange (Asvf) is calculated from Vf.

$$A_{svf} = \frac{V_f}{0.87 f_y}$$

The yield stress of the reinforcement is denoted as fy .

Example 1: A PSC beam span 10m of rectangular section, 120 mm wide and 300mm deep is axially prestressed by a cable carrying an effective force of 180 kN. The beam supports a total UDL of 5kN/m which includes the self weight of the member. Compare the magnitude of the principal tension developed in the beam with and without the axial prestress.

Solution:

A=36x10^3mm2

I=27x10^7

Wd=5kN/m

V=5x10/2=25kN

tv=3V/2bh=3/2x25x10^3/120x300=1.05N/mm2

Principal stresses=+-1/2xsqrt(4xtv2)=+-1.05N/mm2

Fx=180x10^3/36x10^3 =5N/mm2

Maximum and minimum principal stresses= (fx/2)+-(1/2xsqrt(fx2+4tv2))

Maximum=5.23N/mm2

Minimum=-0.23N/mm2

Principal tension reduced by, (1.05-0.23/1.05)=78%

Example 2: For the beam in example 1, instead of axial prestressing a curved cable having an eccentricity of 100mm at the centre of span and reducing to zero at the supports is used, the effective force in the cable being 180kN. Estimate the percentage reduction in the principal tension in compression with the case of axial prestressing.

Solution:

Slope at support=4e/l=4x100/10/1000=0.04radians.

Vertical=180x0.04=7.2kN

Horizontal=180kN

V=25-7.2=17.8kN

Maximum shear stress=3V/2bh=3x17.8x10^3/2x120x300=0.74N/mm2

Fx=180x10^3/120x300=5N/mm2

Fmax,min=(5/2)+-(1/2xsqrt(5^2+4x0.74^2)

Maximum=5.12N/mm2

Minimum =-0.12N/mm2

Percentage reduction(with prestress)=0.23-012/0.23x100=48%

Percentage reduction(without prestress)=1.05-0.12/1.05=88.5%

Example 3: If the beam in the example 2 is additionally prestressed by vertical cables imparting a stress of 2.5N/mm2 in the direction of the depth of the beam, estimate the nature of principal stresses developed at the support section.

Solution:

fy=2.5N/mm2

fx=5N/mm2

tv=0.74N/mm2

fmax,min= (5+2.5/2)+-(1/2)xsqrt((5-2.5)^2+4x0.74^2)

maximum=5.2N/mm2

Minimum=2.3N/mm2

Module -5

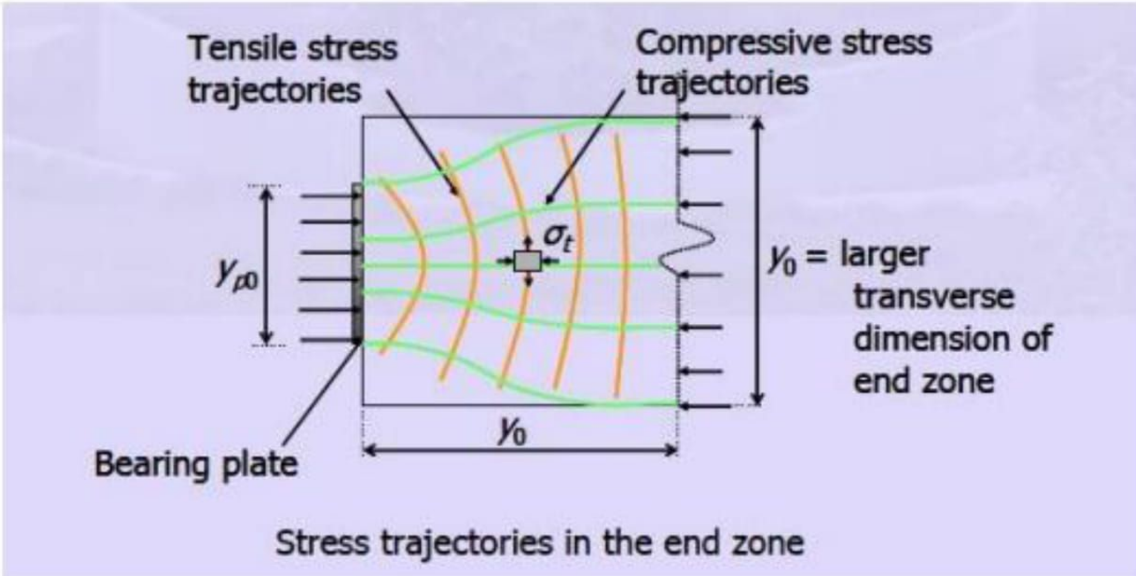
Anchorage zone stresses and design of anchorages. Composite Sections: Types of composite construction - Analysis of composite sections - Deflection –Flexural and shear strength of composite sections.

In prestressed concrete structural members, the prestressing force is usually transferred from the prestressing steel to the concrete in one of two different ways. In post-tensioned construction, relatively small anchorage plates transfer the force from the tendon to the concrete immediately behind the anchorage by bearing. For pretensioned members, the force is transferred by bond between the steel and the concrete. In either case, the prestressing force is transferred in a relatively concentrated fashion, usually at the end of the member, and involves high local pressures and forces. A finite length of the member is required for the concentrated forces to disperse to form the linear compressive stress distribution assumed in design.

The length of member over which this dispersion of stress takes place is called the transfer length (in the case of pretensioned members) and the anchorage length (for post-tensioned members). Within these so-called anchorage zones, a complex stress condition exists.

Transverse tension is produced by the dispersion of the longitudinal compressive stress trajectories and may lead to longitudinal cracking within the anchorage zone. Similar zones of stress exist in the immediate vicinity of any concentrated force, including the concentrated reaction forces at the supports of a member.

The anchorage length in a post-tensioned member and the magnitude of the transverse forces (both tensile and compressive), that act perpendicular to the longitudinal prestressing force, depend on the magnitude of the prestressing force and on the size and position of the anchorage plate or plates. Both single and multiple anchorages are commonly used in post-tensioned construction. A careful selection of the number, size, and location of the anchorage plates can often minimize the transverse tension and hence minimize the transverse reinforcement requirements within the anchorage zone. The stress concentrations within the anchorage zone in a pretensioned.



Stress trajectories in the end zone

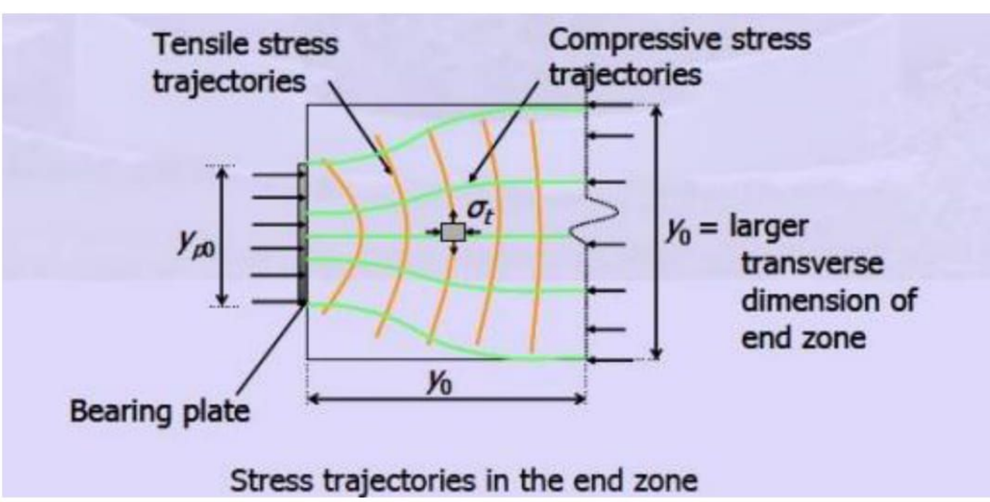
Bursting force: A portion of a pre-stressed member surrounding the anchorage is the end block.

End block

Bursting force

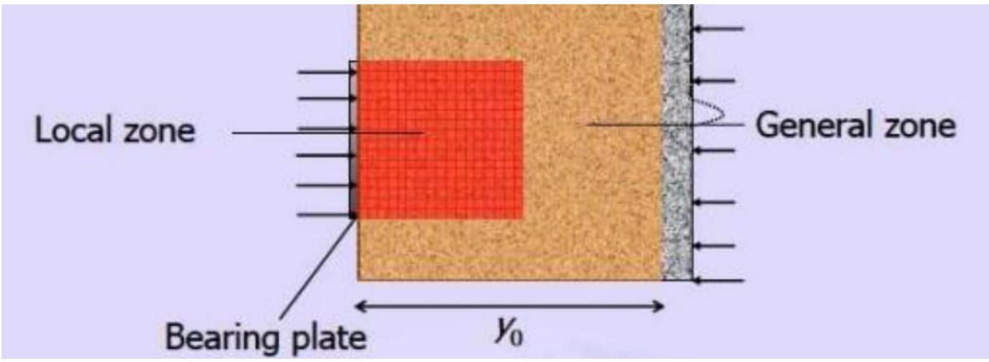
A portion of a pre-stressed member surrounding the anchorage is the end block. Through the length of the end block, pre-stress is transferred from concentrated areas to become linearly distributed fiber stresses at the end of the block. The theoretical length of this block, called the lead length is not more than the height of the beam.

But the stress distribution within this block is rather complicate.



Stress trajectories in the end zone

The larger transverse dimension of the end zone is represented as yo. The corresponding dimension of the bearing plate is represented as ypo. For analysis, the end zone is divided into a local zone and a general zone.

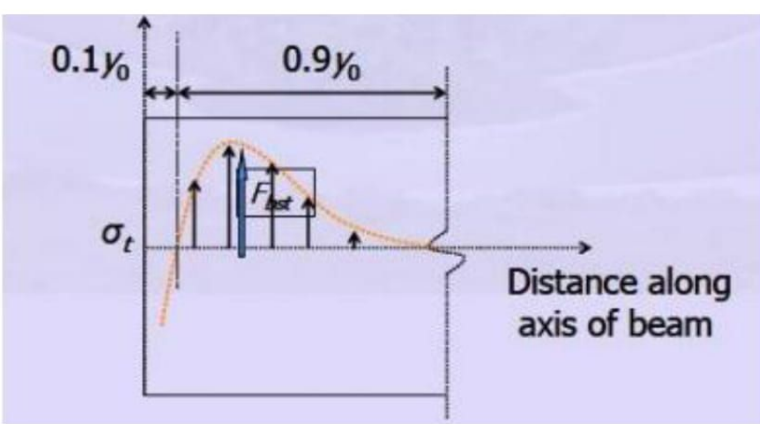


The local zone is the region behind the bearing plate and is subjected to high bearing stress and internal stresses. The behavior of the local zone is influenced by the anchorage device and the additional confining spiral reinforcement.

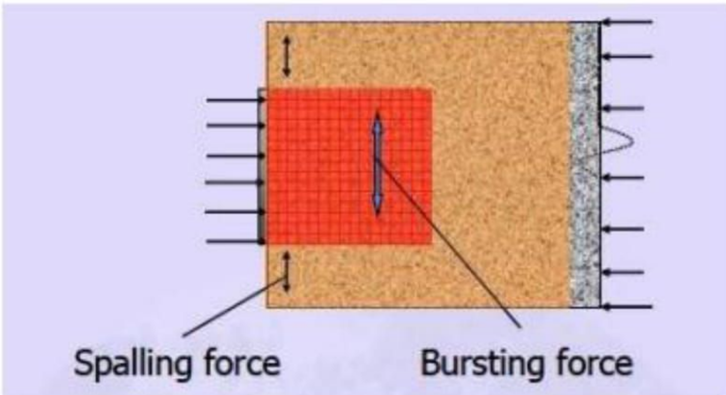
The general zone is the end zone region which is subjected to spalling of concrete. The zone is strengthened by end zone reinforcement.

The transverse stress (?t) at the CGC varies along the length of the end zone. It is compressive for a distance 0.1yo from the end and tensile thereafter, which drops down to zero at a distance yo from the end.

The transverse tensile stress is known as splitting tensile stress. The resultant of the tensile stress in a transverse direction is known as the bursting force (Fbst).



Besides the bursting force there is spalling forces in the general zone.



Fbst for an individual square end zone loaded by a symmetrically placed square bearing plate according to Cl 18.6.2.2 is,

$$F_{bst} = P_K\left[0.32 - 0.3\frac{y_{po}}{y_o}\right]$$

Where, PK = pre-stress in the tendon;

y po = length of a side of bearing plate;

yo = transverse dimension of the end zone.

It can be observed that with the increase in size of the bearing plate the bursting force Fbst reduces.

End Zone reinforcement

Transverse reinforcement - end zone reinforcement or anchorage zone reinforcement or bursting link - is provided in each principle direction based on the value of Fbst. The reinforcement is distributed within a length from 0.1yo to yo from an end of the member.

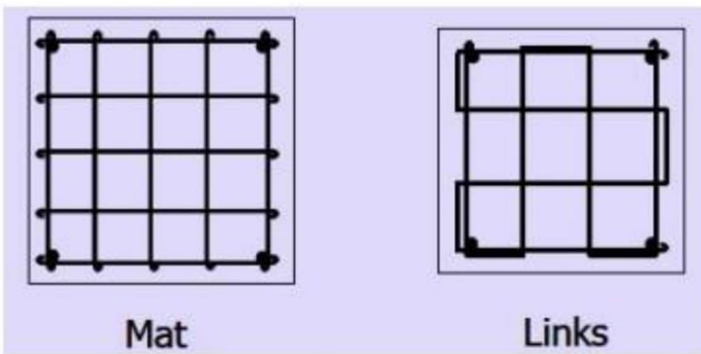
The amount of end zone reinforcement in each direction Ast

The parameter represents the fraction of the transverse dimension covered by the bearing plate.

The stress in the transverse reinforcement, fs = 0.87fy.

When the cover is less than 50 mm, fs = a value corresponding to a strain of 0.001.

The end zone reinforcement is provided in several forms, some of which are proprietary of the construction firms. The forms are closed stirrups, mats or links with loops.



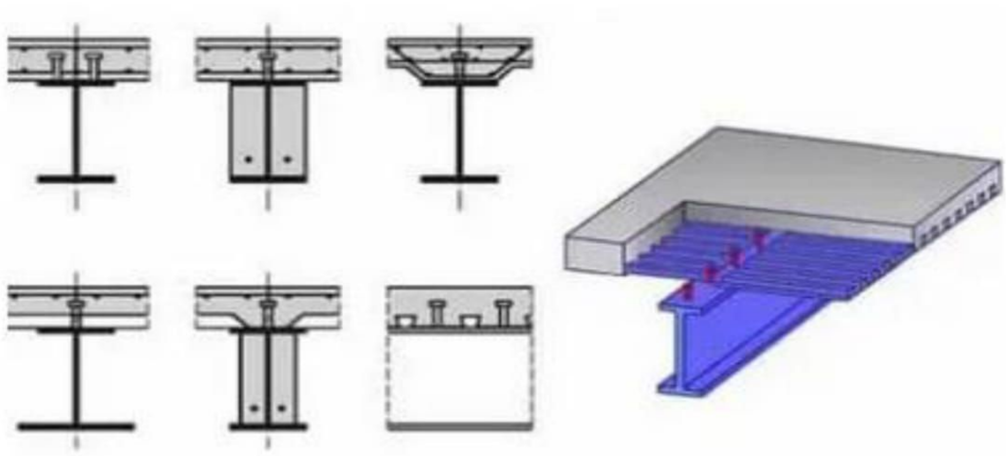
Composite Construction

Many applications of prestressed concrete involve the combination of precast prestressed concrete beams and in situ reinforced concrete slabs. Some examples of such composite construction. An in situ infill between precast beams while an in situ topping . The former type of construction is often used in bridges, while the latter is common in building construction. The beams are designed to act alone under their own weight plus the weight of the wet concrete of the slab. Once the concrete in the slab has hardened and provided that there is adequate horizontal shear connection between them, the slab and beam behave as a composite section under design load. The beams act as permanent formwork for the slab, which provides the compression flange of the composite section. The section size of the beam can thus be kept to a minimum, since a compression flange is only required at the soffit at transfer. This leads to the use of inverted T-, or 'top-hat', sections.

Types of composite construction

Composite Beams

Composite beams are constructed from two or more different types of materials such as steel and concrete, and various valid cross sections have been utilized



Composite beam

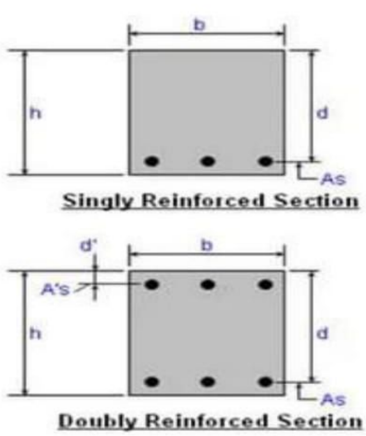
Based on Cross-Section Shapes

Several cross sectional shapes of beams are available and used in different parts of of structures. These beams can be constructed from reinforced concrete, steel, or composite materials:

Reinforced concrete cross sectional shapes include:

Rectangular beam

This type of beam is widely used in the construction of reinforced concrete buildings and other structures.

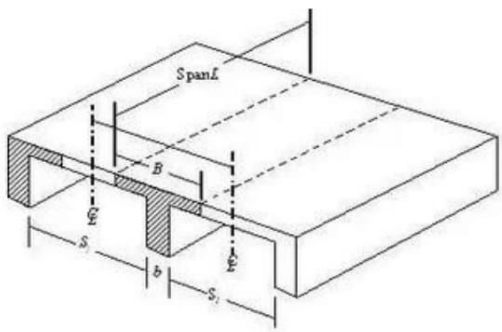


Rectangular Reinforced concrete beam

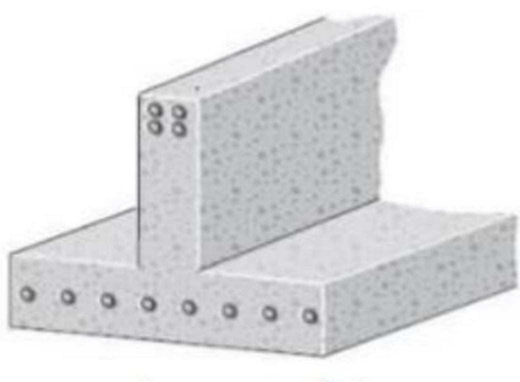
10. T-section beam

This type of beam is mostly constructed monolithically with reinforced concrete slab. Sometimes, Isolated T-beam are constructed to increase the compression strength of concrete.

Added to that, inverted T-beam can also be constructed according to the requirements of loading imposed.



T-beam



Inverted T-beam

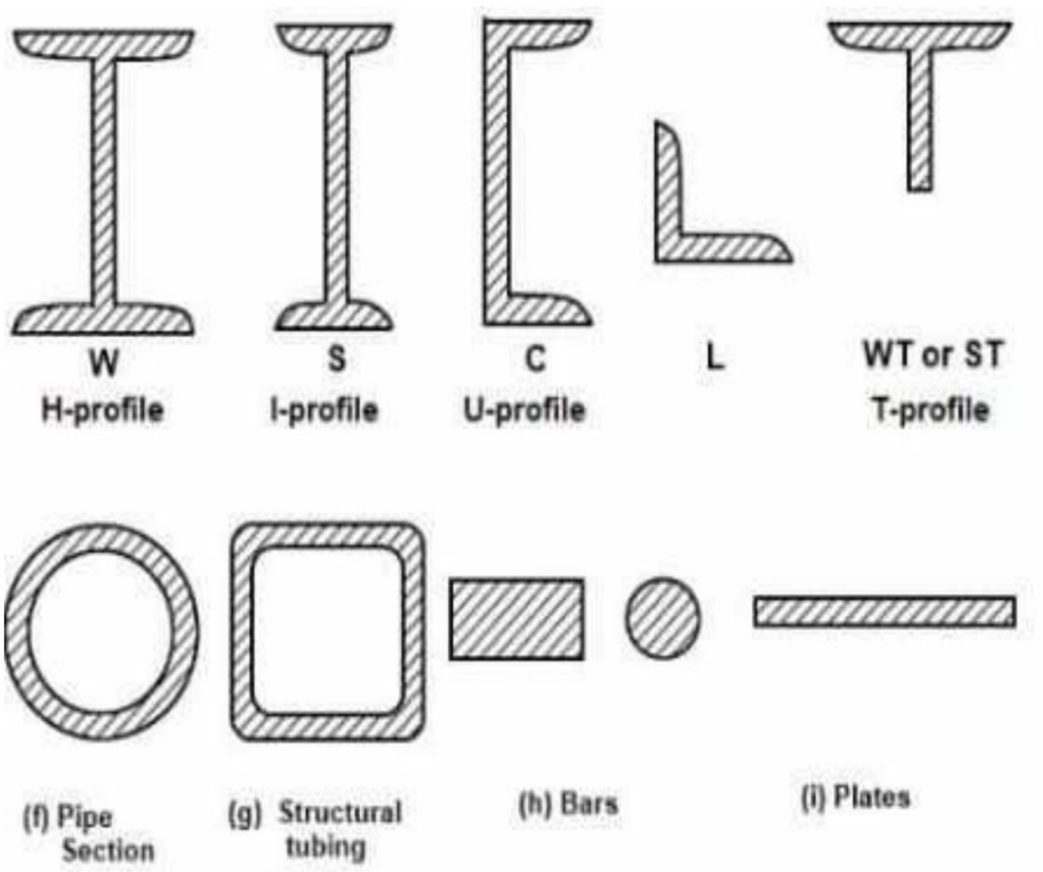
11. L-section beam

This type of beam is constructed monolithically with reinforced concrete slab at the perimeter of the structure, as illustrated in Fig. 10.

Steel cross sectional shapes include:

There are various steel beam cross sectional shapes. Each cross sectional shape offer superior advantages in a given conditions compare with other shapes.

Square, rectangular, circular, I-shaped, T-shaped, H-shaped,C-shaped, and tubular are examples of beam cross sectional shapes constructed from steel.

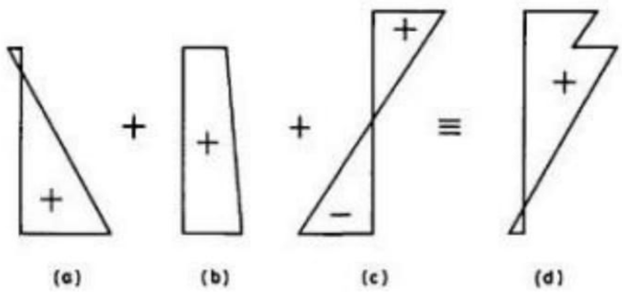


Analysis of composite sections

The stress distributions in the various regions of the composite member are shown in Fig. The stress distribution in Fig. 10.2(a) is due to the self weight of the beam, with the maximum compressive stress at the lower extreme fibre. Once the slab is in place, the stress distribution in the beam is modified to that shown in Fig., where the bending moment at the section, Md is that due to the combined self weight of the beam and slab.

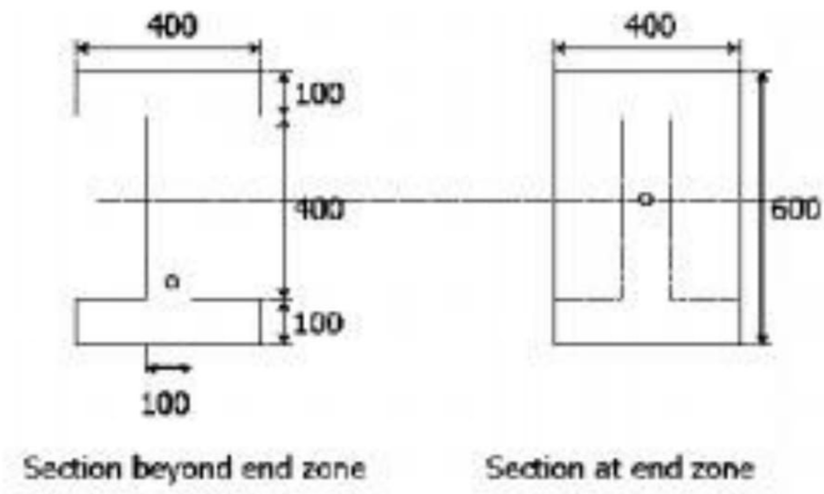
Once the concrete in the slab has hardened and the imposed load acts on the composite section, the additional stress distribution is shown in Fig.. This is determined by ordinary bending theory, but using the composite section properties.

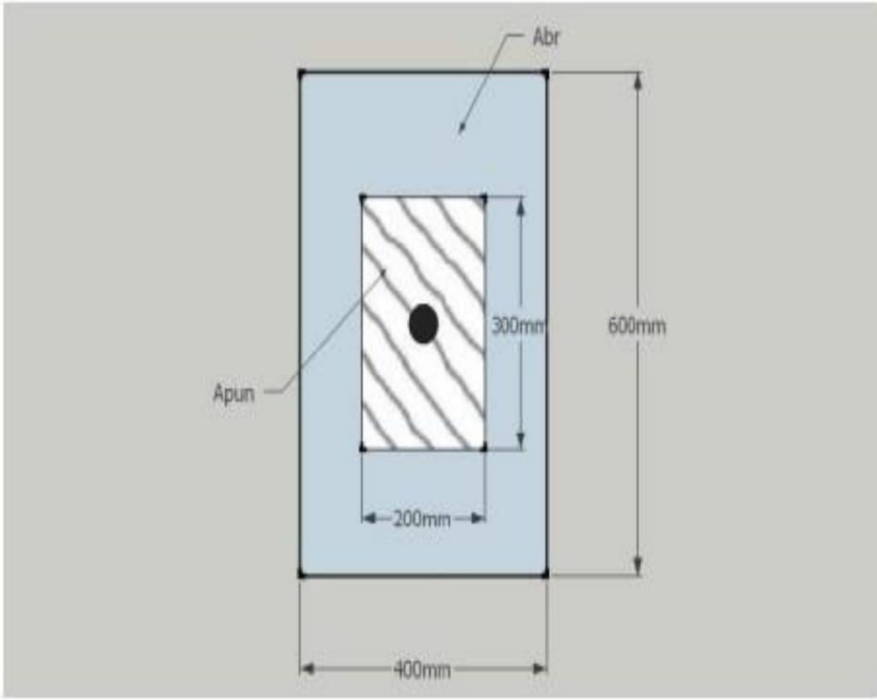
The final stress distribution is shown Figure



Bearing plate & End block

Design the bearing plate and the end zone reinforcement for the following bonded post-tensioned beam. The strength of concrete at transfer is 50 MPa. A pre-stressing force of 1055 kN is applied by a single tendon. There is no eccentricity of the tendon at the ends.





Bearing Plate

Assume area of bearing plate to be 200 mm x 300 mm

$$f_{br} = \frac{P_K}{A_{pun}}$$

P_K = 1055 kN

A_{pun} = 200 x 300 = 60000 mm^2

f_{br} = $\frac{1055 \times 10^3}{60000}$ = 17.58 MPa

A_{br} = 400 x 600 = 240000 mm^2

$$f_{br,all} = 0.48 f_{ci} \sqrt{\frac{A_{br}}{A_{pun}}}$$

$$= 0.48\times 50\sqrt{\frac{240000}{60000}}$$ = 48 MPa

$$\leq 0.8\times f_{ci} = 40 \text{ MPa}$$

$$f_{br} \leq f_{br,all} = 40 MPa$$

End Block

In vertical direction

$$F_{bst} = P_K\left[0.32 - 0.3\frac{y_{po}}{y_o}\right]$$

$$= 1055\left[0.32 - 0.3\frac{300}{600}\right]$$ = 179.35 kN

In horizontal direction

$$F_{bst} = P_K\left[0.32 - 0.3\frac{y_{po}}{y_o}\right]$$

$$= 1055\left[0.32 - 0.3\frac{200}{400}\right]$$ = 179.35 kN

$$A_{st} = \frac{F_{bst}}{0.87 f_y}$$

$$= \frac{179.35\times 10^3}{0.87\times 250}$$ = 824.60 mm^2

Provide 10 mm 2L stirrups in both directions as F_{bst} is same in those

$A_w = \frac{\pi \times 10^2}{4} = 78.54\ mm^2$

No of stirrups $= \frac{824.60}{2 \times 78.54} = 6$ Nos

Provide $\frac{2}{3}^{rd}$ A_{st} from 0.1 y_o = 60 mm to 0.5 y_o = 300 mm and $\frac{1}{3}^{rd}$ A_{st} from 0.5 y_o = 300 mm to y_o = 600 mm, both vertically and horizontal.